YEAR 1 PURE   Chapter 12 DIFFERENTIATION

- First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^3$ then $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

expand + simplify $= \lim_{h \to 0} 3x^2 + 3xh + h^2$

(as $h \to 0$ $3xh = 0$, $h^2 = 0$)

$f'(x) = 3x^2$

- NOTATION

$f(x) \xrightarrow[\text{differentiate}]{} f'(x) \xrightarrow[\text{differentiate}]{} f''(x)$

$y \longrightarrow \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2}$

$4x^3 \xrightarrow[\text{multiply by power then subtract 1 from power}]{} 12x^2 \xrightarrow[\text{multiply by power then subtract 1 from power}]{} 24x$

- Differentiate

$$y = \sqrt{x} = x^{1/2} \quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$y = \frac{1}{3x^3} = \frac{1}{3}x^{-3} \quad \frac{dy}{dx} = -\frac{3}{3}x^{-4}$$

$$y = \frac{2x^3 + 3x}{x} = 2x^2 + 3 \quad \frac{dy}{dx} = 4x$$

both terms get divided by x

- Find the Gradient of a tangent

Find $\frac{dy}{dx}$ then sub in x

- Find max and min points

max point $\frac{d^2y}{dx^2} < 0$

min point $\frac{d^2y}{dx^2} > 0$