

## Topic assessment

1. Find  $y$  in terms of  $x$  given that  $\frac{dy}{dx} = x(y-1)$ . [4]
2. Solve  $(x-1)\frac{dy}{dx} = xy$  for  $x > 1$  and  $y > 0$ , given that  $y = 1$  when  $x = 3$ . [5]
3. Obtain a particular solution to  $(1-e^{2y})\frac{dy}{dx} = e^y$  given that  $y = 0$  when  $x = 2$ .  
(There is no need to express  $y$  in terms of  $x$ ). [5]
4. Find an expression for  $y$  in terms of  $x$  given that  $x^2\frac{dy}{dx} - y^2 = 0$ . [4]
5. At time  $t$  seconds the rate of increase in the concentration of flesh eating bugs in a controlled environment is proportional to the concentration  $C$  of bugs present. Initially  $C = 100$  bugs and after 2 seconds there are five times as many.
  - (i) Write down a differential equation connecting  $\frac{dC}{dt}$ ,  $C$  and  $t$  and hence find an expression for  $C$  in terms of  $t$ . [7]
  - (ii) How many bugs are present after 5 seconds? [2]
  - (iii) When will the number of bugs exceed 5000? [3]
  - (iv) Find the time at which the concentration of bugs has increased by 50% of the initial concentration. [3]
6. Water is pouring out of a small hole in the bottom of a conical container of height 25 cm. Initially the container is full. The rate at which the height  $x$  of the water remaining in the container is given by  $\frac{dx}{dt} = -\frac{50}{\pi}x^{-\frac{3}{2}}$ .
  - (i) Solve the differential equation to find  $x$  in terms of  $t$ . [5]
  - (ii) How long does it take for the container to empty completely? [2]

Total 40 marks