Q1.

The numbers α, β and γ satisfy the equations

$$\alpha^2 + \beta^2 + \gamma^2 = -10 - 12\mathrm{i}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 5 + 6\mathrm{i}$$

(a) Show that $\alpha + \beta + \gamma = 0$.

(2)

(b) The numbers α, β and γ are also the roots of the equation

$$z^3 + pz^2 + qz + r = 0$$

Write down the value of p and the value of q.

(2)

(c) It is also given that $\alpha = 3\mathrm{i}$.

(i) Find the value of r.

(3)

(ii) Show that β and γ are the roots of the equation

$$z^2 + 3\mathrm{i}z - 4 + 6\mathrm{i} = 0$$

(2)

(iii) Given that β is real, find the values of β and γ.

(3)

(Total 12 marks)

Q2.

The cubic equation

$$z^3 + pz + q = 0$$

has roots α , β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$.

(1)

(ii) Express $\alpha\beta\gamma$ in terms of q.

(1)

(b) Show that

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

(3)

(c) Given that $\alpha = 4 + 7i$ and that p and q are real, find the values of :

(i) β and γ ;

(2)

(ii) p and q.

(3)

(d) Find a cubic equation with integer coefficients which as roots, $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

(3)
(Total 13 marks)

Q3.

The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

(2)

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$.

(2)

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$

(5)
(Total 9 marks)

Q4.

The cubic equation

$$z^3 + pz^2 + qz + 37 - 36\text{i} = 0$$

where p and q are constants, has three complex roots, α, β and γ. It is given that $\beta = -2 + 3\text{i}$ and $\gamma = 1 + 2\text{i}$.

(a) (i) Write down the value of $\alpha\beta\gamma$.

(1)

(ii) Hence show that $(8 + \text{i})\alpha = 37 - 36\text{i}$.

(2)

(iii) Hence find α, giving your answer in the form $m + n\text{i}$, where m and n are integers.

(3)

(b) Find the value of p.

(1)

(c) Find the value of the complex number q.

(2)
(Total 9 marks)

Q5.

The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$.

(2)

(ii) Hence find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$.

(2)

(b) The value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is −4.

(i) Explain why α, β and γ cannot all be real.

(1)

(ii) By considering $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, find the possible values of k.

(4)
(Total 9 marks)

Q6.

The equation

$$2x^2 + 3x - 6 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

(2)

(b) Hence show that $\alpha^3 + \beta^3 = -\frac{135}{8}$.

(3)

(c) Find a quadratic equation, with integer coefficients, whose roots are $\alpha + \frac{\alpha}{\beta^2}$ and $\beta + \frac{\beta}{\alpha^2}$.

(6)
(Total 11 marks)

Q7.

The cubic equation

$z^3 - 4iz^2 + qz - (4 - 2i) = 0$

where q is a complex number, has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha + \beta + \gamma$;

(1)

(ii) $\alpha\beta\gamma$.

(1)

(b) Given that $\alpha = \beta + \gamma$, show that:

(i) $\alpha = 2i$;

(1)

(ii) $\beta\gamma = -(l + 2i)$;

(2)

(iii) $q = -(5 + 2i)$.

(3)

(c) Show that β and γ are the roots of the equation

$$z^2 - 2iz - (1 + 2i) = 0$$

(2)

(d) Given that β is real, find β and γ.

(3)

(Total 13 marks)

Q8.

The equation $x^3 - 8x^2 + cx + d = 0$, where c and d are real numbers, has roots α, β, γ.

When plotted on an Argand diagram, the triangle with vertices at α, β, γ has an area of 8.

Given $\alpha = 2$, find the values of c and d.

Fully justify your solution.

(Total 5 marks)

Q9.

The roots of the quadratic equation

$$x^2 + 2x - 5 = 0$$

are α and β

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

(2)

(b) Calculate the value of $\alpha^2 + \beta^2$.

(2)

(c) Find a quadratic equation which has roots $\alpha^3\beta + 1$ and $\alpha\beta^3 + 1$.

(5)
(Total 9 marks)

Q10.

α, β and γ are the real roots of the cubic equation

$$x^3 + mx^2 + nx + 2 = 0$$

By considering $(\alpha - \beta)^2 + (\gamma - \alpha)^2 + (\beta - \gamma)^2$, prove that

$$m^2 \geq 3n$$

(Total 4 marks)

Q11.

The equation

$x^2 - 4x + 13 = 0$

has roots α and β.

(a) (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$.

(2)

(ii) Deduce that $\alpha^2 + \beta^2 = -10$.

(2)

(iii) Explain why the statement $\alpha^2 + \beta^2 = -10$ implies that α and β cannot both be real.

(2)

(b) Find in the form $p + \mathrm{i}q$ the values of:

(i) $(\alpha + \mathrm{i}) + (\beta + \mathrm{i})$;

(1)

(ii) $(\alpha + \mathrm{i})(\beta + \mathrm{i})$.

(2)

(c) Hence find a quadratic equation with roots $(\alpha + \mathrm{i})$ and $(\beta + \mathrm{i})$.

(2)
(Total 11 marks)