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KS5 "Full Coverage": Differentiation (Yr2)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Use the chain rule (and differentiate natural logs).

[Edexcel C3 June 2011 Q1a] Differentiate with respect to x

$$ln\,(x^2 + 3x + 5)$$

$\frac{d\left(ln\,(x^2+3x+5)\right)}{dx}$ ....

Question 2

Categorisation: Use the product rule (and differentiate e^x)

[Edexcel C3 June 2007 Q3a] A curve C has equation $y = x^2 e^x$.

Find $\frac{dy}{dx}$, using the product rule for differentiation.

$\frac{dy}{dx} =$ ....

Question 3

Categorisation: Use the chain rule combined with product rule.

*[Edexcel C3 June 2009 Q4ia]*Differentiate with respect to x

$$x^2\,cos\,3x$$

$\frac{d(x^2\,cos\,3x)}{dx} =$ ....

Question 4

Categorisation: Use the quotient rule.

[Edexcel C3 June 2016 Q2a]

$$y = \frac{4x}{x^2 + 5}$$

Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

$\frac{dy}{dx} =$ ....

Question 5

Categorisation: As above.

[Edexcel C3 June 2013(R) Q5a]

Differentiate $\frac{\cos 2x}{\sqrt{x}}$ with respect to x .

....

Question 6

Categorisation: Further practice of product with chain rule.

[Edexcel A2 SAM P2 Q3]

Given $y = x(2x+1)^4$, show that

$$\frac{dy}{dx} = (2x+1)^n(Ax+B)$$

where n , A and B are constants to be found.

....

Question 7

Categorisation: Differentiate trig functions raised to a power.

[Edexcel C3 June 2005 Q2ai]

Differentiate with respect to x

$$3\sin^2 x + \sec 2x$$

$\frac{d}{dx}(3\sin^2 x + \sec 2x) =$ ....

Question 8

Categorisation: As above, but with a multiple of x.

[Edexcel C3 June 2013(R) Q5b]

Show that $\frac{d}{dx}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant to be found.

$\mu =$ ....

Question 9

Categorisation: Appreciate that $ln(kx)$ differentiates to $\frac{1}{x}$ regardless of k.

[Edexcel C3 June 2012 Q7ai]

Differentiate with respect to x ,

$$x^{\frac{1}{2}} \ln(3x)$$

$\frac{d\left(x^{\frac{1}{2}} \ln(3x)\right)}{dx} =$ ....

Question 10

Categorisation: Appreciate that $\frac{1}{f(x)g(x)}$ can be written as $f(x)^{-1}g(x)^{-1}$ before differentiating.

[Edexcel C3 June 2013 Q5c]

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Find an expression for $\frac{d^2y}{dx^2}$ in terms of x . Give your answer in its simplest form.

$\frac{d^2y}{dx^2} =$ ....

Question 11

Categorisation: Differentiate in a modelling context.

[Edexcel C3 June 2006 Q4c]

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T, t minutes after it enters the liquid, is given by

$T = 400e^{-0.05t} + 25\ ,\ t \geq 0$

Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$

.... $^{\circ}C$ /min

Question 12

Categorisation: As above.

[Edexcel C3 June 2017 Q8b]

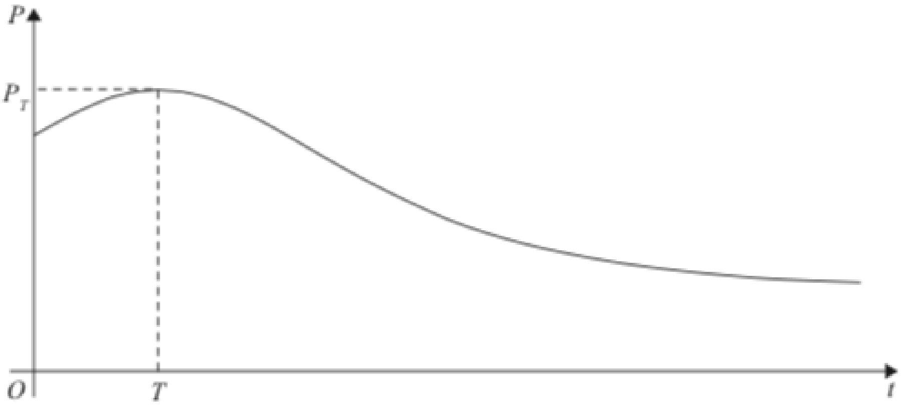

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1+3e^{-0.9t}} + 40\,, \qquad t \in \mathbb{R}\,,\ \ t \geq 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

Find $\frac{dP}{dt}$

$\frac{dP}{dt} =$ ....

Question 13

Categorisation: Use the fact that $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$

[Edexcel C3 June 2017 Q7ii]

Given $x = \ln(\sec 2y)\,, \qquad 0 < y < \frac{\pi}{4}$

find $\frac{dy}{dx}$ as a function of x in its simplest form.

$\frac{dy}{dx} =$ ....

Question 14

Categorisation: Determine $\frac{dy}{dx}$ for expressions of the form $x = f(y)$ where f is a trigonometric function. Note that the specification effectively expects students to differentiate expressions such as $arcsin\ x$, even though questions wouldn't explicitly ask as such.

[Edexcel C3 June 2013(R) Q5c]

Given $x = 2\ sin\ \left(\frac{y}{3}\right)$, find $\frac{dy}{dx}$ in terms of x , simplifying your answer.

$\frac{dy}{dx} =$ ....

Question 15

Categorisation: As above.

[Edexcel C3 June 2016 Q5ii]

Given $x = sin^2 2y\ ,\ 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y .

Write your answer in the form

$\frac{dy}{dx} = p\ cosec\ (qy)\ ,\ 0 < y < \frac{\pi}{4}$,

where p and q are constants to be determined.

....

Question 16

Categorisation: Find the range of values for which a function is increasing or decreasing.

[Edexcel C3 June 2016 Q2b Edited]

$$y = \frac{4x}{x^2 + 5}$$

$$\frac{dy}{dx} = \frac{20 - 4x^2}{(x^2 + 5)^2}$$

Hence find the set of values of x for which $\frac{dy}{dx} < 0$.

....

Question 17

Categorisation: Use differentiation to determine a turning point.

[Edexcel C3 June 2016 Q5i]

Find, using calculus, the x coordinate of the turning point of the curve with equation

$y = e^{3x} \cos 4x \, , \frac{\pi}{4} \leq x < \frac{\pi}{2}$

Give your answer to 4 decimal places.

....

Question 18

Categorisation: Use differentiation in the context of numerical methods.

[Edexcel A2 Specimen Papers P2 Q6b Edited]

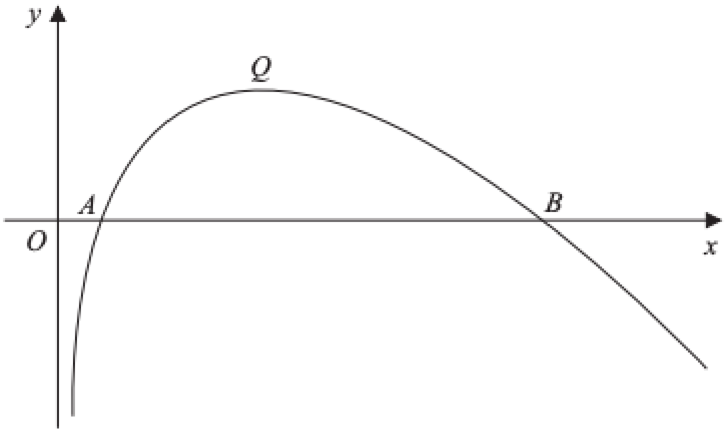

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$f(x) = (8 - x)\ ln\, x\ ,\ x > 0$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 2.

Show that the x coordinate of Q satisfies

$$x = \frac{a}{1 + \ ln\, x}$$

where a is a constant to be found.

....

Question 19

Categorisation: Determine a turning point with a more difficult expression.

[Edexcel A2 SAM P1 Q15a Edited]

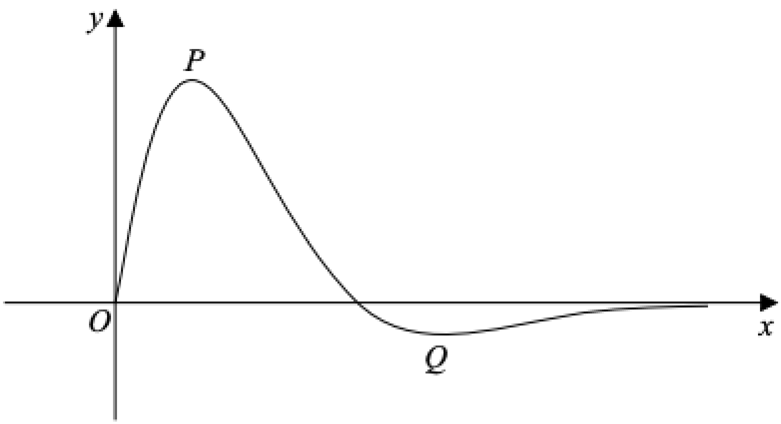

Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$, where

$f(x) = \frac{4 \sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leq x \leq \pi$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

Show that the x coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{a}$, where a is a constant to be found.

....

Question 20
Categorisation: As above.

[Edexcel C3 June 2013 Q4a]

$f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$

One of the turning points of the curve with equation $y = f(x)$ is $(0, -16)$. Using calculus, find the exact coordinates of the other turning point.

....

Question 21
Categorisation: Determine the equation of a tangent to the curve.

[Edexcel C3 June 2009 Q4ii]

A curve *C* has the equation

$y = \sqrt{4x+1}, \ \ x > -\frac{1}{4}, \ \ y > 0$

The point *P* on the curve has x -coordinate 2. Find an equation of the tangent to *C* at *P* in the form $ax + by + c = 0$, where a, b and c are integers.

....

Question 22

Categorisation: As above, but where $\boldsymbol{x = f(y)}$

[Edexcel C3 June 2014 Q3b]

The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$. Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π .

....

Question 23

Categorisation: Determine the equation of a normal to the curve.

[Edexcel C3 June 2010 Q2]

A curve *C* has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}$$

The point *P* on *C* has x -coordinate 2.

Find an equation of the normal to *C* at *P* in the form $ax + by + c = 0$, where a , b and c are integers.

....

Question 24

Categorisation: As above.

[Edexcel C3 June 2011 Q7b Edited]

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

$$x \neq \pm 3, x \neq -\frac{1}{2}$$

The curve *C* has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on *C*.

Find an equation of the normal to *C* at *P*.

....

Question 25

Categorisation: As above, but in the context of having done algebraic long division first.

[Edexcel C3 June 2016 Q6b Edited]

$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, x > 2, x \in \mathbb{R}$$

Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + 3 + \frac{4}{x-2}$$

Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$.

....

Question 26

Categorisation: Determine the gradient of parametric equations.

[Edexcel C4 June 2017 Q1a]

The curve C has parametric equations

$x = 3t - 4 \qquad y = 5 - \frac{6}{t} \qquad t > 0$

Find $\frac{dy}{dx}$ in terms of t .

$\frac{dy}{dx} =$ ....

Question 27

Categorisation: As above, but with trigonometric expressions.

[Edexcel A2 SAM P1 Q13a]

The curve C has parametric equations $x = 2\cos t$, $y = \sqrt{3}\cos 2t$, $0 \leq t \leq \pi$

Find an expression for $\frac{dy}{dx}$ in terms of t .

$\frac{dy}{dx} =$ ....

Question 28

Categorisation: Determine a particular gradient at a point on a parametric curve.

[Edexcel C4 June 2013(R) Q7a]

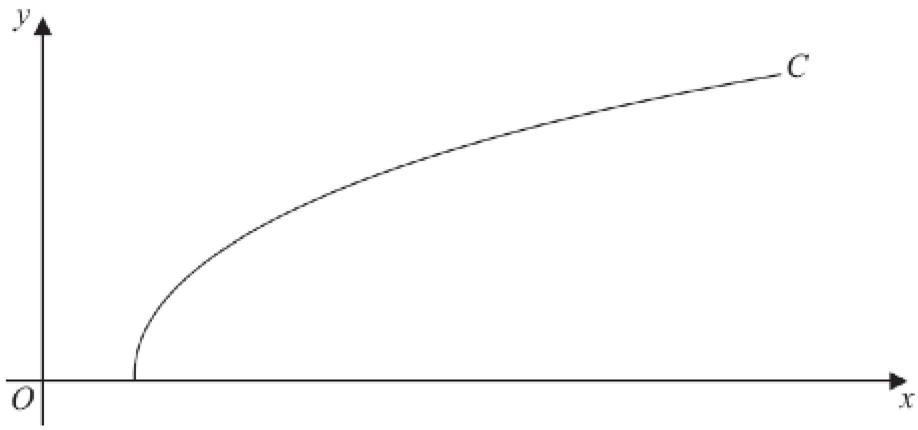

Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$x = 27\, sec^{\,3} t\ ,\quad y = 3\, tan\, t\ ,\quad 0 \leq t \leq \frac{\pi}{3}$

Find the gradient of the curve *C* at the point where $t = \frac{\pi}{6}$.

……………………

Question 29

Categorisation: Determine the equation of a normal to a parametric curve.

[Edexcel C4 Jan 2011 Q6a]

The curve *C* has parametric equations

$x = \ ln\, t\ ,\quad y = t^2 - 2\ ,\quad t > 0\ .$

Find an equation of the normal to *C* at the point where $t = 3$.

……………………

Question 30

Categorisation: As above, but involving exponential terms of the form a^t.

[Edexcel C4 Jan 2013 Q5c Edited]

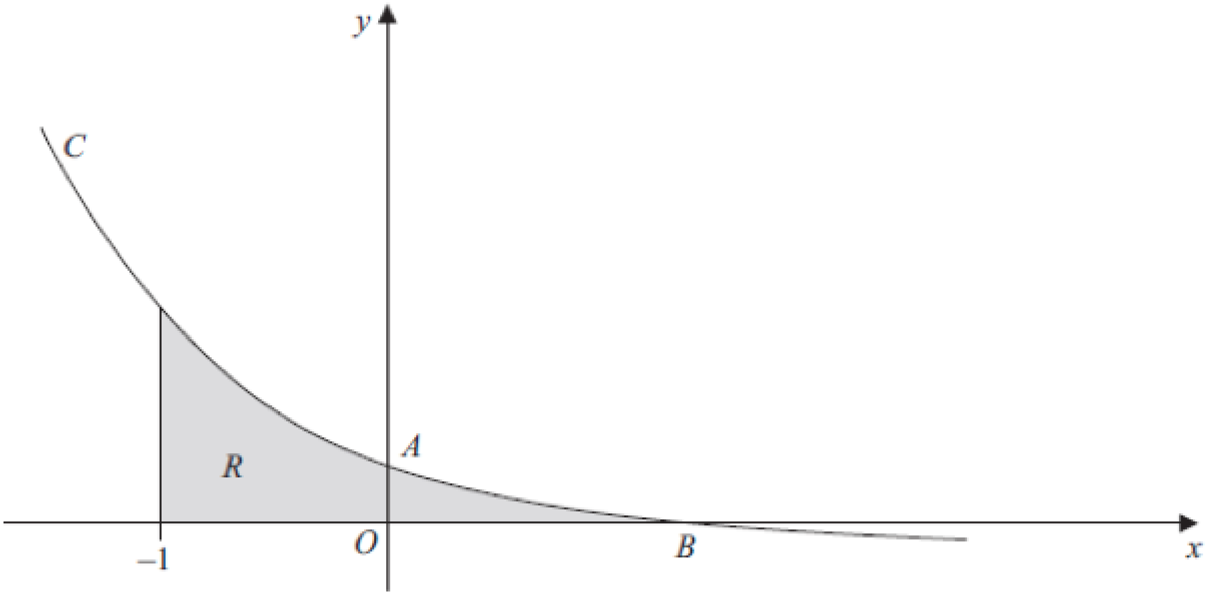

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$.

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

The point A has coordinates $(0,3)$.

Find an equation of the normal to C at the point A.

....

Question 31

Categorisation: Use implicit differentiation.

[Edexcel C4 June 2016 Q3a] The curve *C* has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

$\frac{dy}{dx} =$ ....

Question 32

Categorisation: Use implicit differentiation to find the gradient at a specific point.

[Edexcel C4 June 2017 Q4a]

The curve *C* has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point *P* with coordinates (–2, 4) lies on *C*.

Find the exact value of $\frac{dy}{dx}$ at the point *P*.

$\frac{dy}{dx} =$ ....

Question 33

Categorisation: As above, but involving an exponential term.

[Edexcel C4 June 2013(R) Q2]

The curve *C* has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point $(1,3)$ on the curve *C* can be written in the form $\frac{1}{\lambda}\ ln\ (\mu e^3)$, where λ and μ are integers to be found.

$\frac{dy}{dx} =$ ....

Question 34

Categorisation: Determine a turning point on a parametric curve by solving simultaneously with the original equation.

[Edexcel C4 June 2015 Q2b Edited]

The curve *C* has equation $x^2 - 3xy - 4y^2 + 64 = 0$

The gradient function $\frac{dy}{dx}$ can be expressed as

$$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$$

Find the coordinates of the points on *C* where $\frac{dy}{dx} = 0$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

....

Question 35
Categorisation: As above.

[Edexcel C4 June 2014(R) Q3b Edited]

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

An expression for $\frac{dy}{dx}$ can be expressed as

$$\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$$

Find the values of y for which $\frac{dy}{dx} = 0$.

....

Question 36
Categorisation: Connect different rates of change using the chain rule.

[Edexcel C4 June 2012 Q2b Edited]

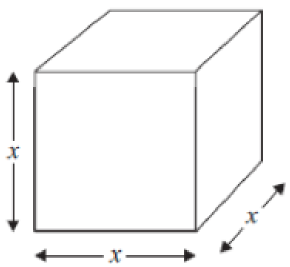

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated. At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1}, find $\frac{dx}{dt}$ when $x = 8$.

$\frac{dx}{dt} =$ ....

Question 37

Categorisation: As above.

[Edexcel C4 June 2012 Q2c Edited]

(Continued from above) Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1}, find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

$\frac{dS}{dt} =$ ....

Question 38

Categorisation: As above.

[Edexcel C4 June 2014 Q4]

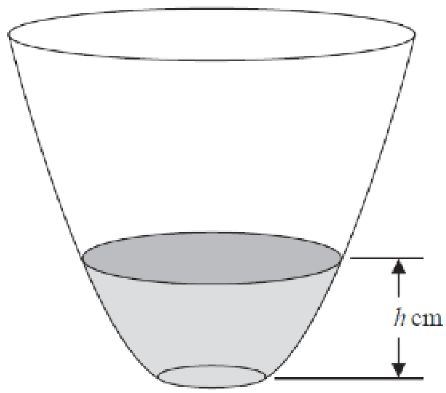

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase. When the depth of the water is h cm, the volume of water V cm^3 is given by

$V = 4\pi h(h + 4)$, $0 \leq h \leq 25$

Water flows into the vase at a constant rate of $80\,\pi\, cm^{\,3} s^{-1}$.

Find the rate of change of the depth of the water, in $cm\ s^{-1}$, when $h = 6$.

$\frac{dh}{dt} =$ .... cms^{-1}

Answers

Question 1

(a)	$\frac{1}{(x^2+3x+5)} \times \ldots , = \frac{2x+3}{(x^2+3x+5)}$	M1,A1

Question 2

(a)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1

Question 3

(i)(a)	$y = x^2 \cos 3x$ Apply product rule: $\left\{\begin{matrix} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{matrix}\right\}$		
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	A1

Question 4

2(a)	$y = \frac{4x}{(x^2+5)} \Rightarrow \left(\frac{dy}{dx}\right) = \frac{4(x^2+5) - 4x \times 2x}{(x^2+5)^2}$	M1A1
	$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{20-4x^2}{(x^2+5)^2}$	M1A1

Question 5

5.(a)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$	B1
	Applies $\frac{vu' - uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $= \frac{\sqrt{x} \times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1A1
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	

Question 6

Attempts the product and chain rule on $y = x(2x+1)^4$	M1
$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1
Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1
$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1

Question 7

(i) $6\sin x\cos x + 2\sec 2x\tan 2x$ or $3\sin 2x + 2\sec 2x\tan 2x$	[M1 for 6 sinx]	M1A1A1 (3)

Question 8

(b)	$\frac{d}{dx}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x\tan 3x \; (= 6\sec^2 3x\tan 3x)$	M1
	$= 6(1+\tan^2 3x)\tan 3x$	dM1
	$= 6(\tan 3x + \tan^3 3x)$	A1

Question 9

(a)(i)	$\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x) \times \frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x}$	M1A1

Question 10

(c)	$\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} \quad = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1

Question 11

(c)	$\frac{dT}{dt} = -20\ e^{-0.05\ t}$	(M1 for $k\,e^{-0.05\ t}$)	M1 A1
	At $t = 50$, rate of decrease $= (\pm)$ 1.64 °C/min		A1

Question 12

	$\frac{d}{dt}e^{kt} = Ce^{kt}$	M1
(b)	$\frac{dP}{dt} = \frac{(1+3e^{-0.9t}) \times -10e^{-0.1t} - 100e^{-0.1t} \times -2.7e^{-0.9t}}{(1+3e^{-0.9t})^2}$	M1 A1

Question 13

(ii)	$x=\ln(\sec 2y) \Rightarrow \frac{dx}{dy}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$	B1
	$\Rightarrow \frac{dy}{dx}=\frac{1}{2\tan 2y}=\frac{1}{2\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{e^{2x}-1}}$	M1 M1 A1

Question 14

(c)	$x=2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy}=\frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1
	$\frac{dy}{dx}=\frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)}=\frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}}=\frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1
	$\frac{dy}{dx}=\frac{3}{\sqrt{4-x^2}}$ cao	A1

Question 15

(ii)	$x=\sin^2 2y \Rightarrow \frac{dx}{dy}=2\sin 2y\times 2\cos 2y$	M1A1
	Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy}=2\sin 4y \Rightarrow \frac{dy}{dx}=\frac{1}{2\sin 4y}=\frac{1}{2}\text{cosec}4y$	M1A1

Question 16

(b)	$\frac{20-4x^2}{\left(x^2+5\right)^2}<0 \Rightarrow x^2>\frac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5},\ x>\sqrt{5}$ or equivalent	dM1A1

Question 17

5 (i)	$y=e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right)=\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x$	M1A1
	Sets $\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x=0 \Rightarrow 3\cos 4x-4\sin 4x=0$	M1
	$\Rightarrow x=\frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x=$ awrt 0.9463 4dp	A1

Question 18

Complete strategy of setting $f'(x)=0$ and rearranges to make $x=\ldots$	M1	3.1a
$\left\{\begin{matrix} u=(8-x) & v=\ln x \\ \frac{du}{dx}=-1 & \frac{dv}{dx}=\frac{1}{x}\end{matrix}\right\}$		
$f'(x)=-\ln x+\frac{8-x}{x}$	M1	1.1b
	A1	1.1b
$-\ln x+\frac{8-x}{x}=0 \Rightarrow -\ln x+\frac{8}{x}-1=0$ $\Rightarrow \frac{8}{x}=1+\ln x \Rightarrow x=\frac{8}{1+\ln x}$ *	A1*	2.1

Question 19

Attempts to differentiate using the quotient rule or otherwise	M1
$f'(x)=\dfrac{e^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1
Sets $f'(x)=0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1
Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ *	A1*

Question 20

4(a)	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.		M1A1
	Puts $f'(x)=0$ to give $x=-1$ and $x=0$ or one coordinate		dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$	CSO	A1

Question 21

(ii)	$y=\sqrt{4x+1},\ x>-\frac{1}{4}$		
	At P, $y=\sqrt{4(2)+1}=\underline{\sqrt{9}}=\underline{3}$	At P, $y=\underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\dfrac{dy}{dx}=\underline{\dfrac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\dfrac{dy}{dx}=\dfrac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\dfrac{dy}{dx}=\dfrac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x=2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence m(**T**) $=\dfrac{2}{3}$		
	Either **T**: $y-3=\frac{2}{3}(x-2)$; or $y=\frac{2}{3}x+c$ and $3=\frac{2}{3}(2)+c \Rightarrow c=3-\frac{4}{3}=\frac{5}{3}$;	$y-y_1=m(x-2)$ or $y-y_1=m(x-\text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y=mx+c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y-9=2(x-2)$; **T**: $3y-9=2x-4$		
	T: $\underline{2x-3y+5=0}$	$\underline{2x-3y+5=0}$ Tangent must be stated in the form $ax+by+c=0$, where a, b and c are integers.	A1

Question 22

(b)	$\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\frac{dx}{dy} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y - \frac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	A1

Question 23

At P, $y = \underline{3}$	B1
$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1<u>A1</u>
$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
$m(\mathbf{N}) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
N: $y - 3 = \frac{1}{18}(x-2)$	M1
N: $\underline{x - 18y + 52 = 0}$	A1

Question 24

(b)	$f(x) = \frac{5}{2x^2+7x+3}$	
	$f'(x) = \frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'(-1) = -\frac{15}{4}$	M1A1
	Uses m_1m_2=-1 to give gradient of normal=$\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)} = their\ \frac{4}{15}$	M1
	$y + \frac{5}{2} = \frac{4}{15}(x+1)$ or any equivalent form	A1

Question 25

(b)	$f'(x) = 2x - \frac{4}{(x-2)^2}$	M1A1ft
	Subs $x = 3$ into $f'(x=3) = 2\times 3 - \frac{4}{(3-2)^2} = (2)$	M1
	Uses $m = -\frac{1}{f'(3)} = \left(-\frac{1}{2}\right)$ with $(3, f(3)) = (3,16)$ to form eqn of normal	
	$y - 16 = -\frac{1}{2}(x-3)$ or equivalent cso	M1A1

Question 26

(a)	$\frac{dx}{dt} = 3,\ \frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3}\ \left\{ = \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2} \right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t **or** their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw

Question 27

Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1
$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t}\quad \left(= 2\sqrt{3}\cos t\right)$	A1

Question 28

(a)	$\frac{dx}{dt} = 81\sec^2 t\sec t\tan t,\ \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t\tan t}\ \left\{ = \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t} \right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\ \left\{ = \frac{3}{54} = \frac{1}{18} \right\}$	$\frac{4}{72}$	A1 **cao cso**

Question 29

(a)	$\frac{dx}{dt} = \frac{1}{t},\quad \frac{dy}{dt} = 2t$	
	$\frac{dy}{dx} = 2t^2$	M1 A1
	Using $mm' = -1$, at $t = 3$	
	$m' = -\frac{1}{18}$	M1 A1
	$y - 7 = -\frac{1}{18}(x - \ln 3)$	M1 A1

Question 30

(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = \text{"2"}$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = \text{"2"}$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso

Question 31

(a) Way 1	$\left\{\frac{dy}{dx}\times\right\}\ \left(4xy + 2x^2\frac{dy}{dx}\right) + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$		M1 A1 B1
	$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{dy}{dx} =\right\}\ \frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or $\frac{4xy+2}{-2x^2-4-\pi\sin(\pi y)}$	Correct answer or equivalent	A1 cso

Question 32

(a) Way 1	$\left\{\frac{dy}{dx}\times\right\}\ 8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} = 0$		M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4\ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 cso

Question 33

$3^{x-1} + xy - y^2 + 5 = 0$		
	$3^{x-1} \to 3^{x-1}\ln 3$	B1 oe
$\left\{\frac{dy}{dx}\times\right\}$ (ignore) $\ 3^{x-1}\ln 3 + \left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
	$xy \to +y + x\frac{dy}{dx}$	B1
	$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	A1
$\{(1,3) \Rightarrow\}\ 3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression.	dM1*
$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5\frac{dy}{dx}$		
$\frac{dy}{dx} = \frac{3+\ln 3}{5}$		dM1*
$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 cso

Question 34

(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ 2x - 3y = 0$		M1
	$y = \frac{2}{3}x$	$x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$	$\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$	$\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}\ x = \frac{24}{5}$ or $-\frac{24}{5}$	$\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}\ y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 cso
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$	When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$	cso	ddM1 A1

Question 35

(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\quad x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y-5)^2 + y^2 + 10(2y-5) + 2y - 4(2y-5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35\ \{= 0\}$ **see notes**	A1 oe
	$(3y-7)(y-5) = 0$ and $y = \ldots$	Method mark for solving a quadratic equation.	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\}\ \frac{7}{3}, 5$	A1 **cao**

Question 36

(b) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$		M1
At $x = 8$		
$\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025\quad (\text{cm s}^{-1})$	2.5×10^{-4}	A1

Question 37

(c) $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	B1
$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x\left(\frac{0.048}{3x^2}\right)$	M1
At $x = 8$	
$\frac{dS}{dt} = 0.024\quad (\text{cm}^2\,\text{s}^{-1})$	A1

Question 38

$\frac{dV}{dt} = 80\pi,\ V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,		
$\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$ $8\pi h + 16\pi$	M1 A1
$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$	$\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	M1 oe
When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi\ \left\{= \frac{80\pi}{64\pi}\right\}$	**dependent on the previous M1** **see notes**	dM1
$\frac{dh}{dt} = \underline{1.25}\ (\text{cm s}^{-1})$	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe