S1 Discrete Random Variables Questions ANSWERS (88 marks)

June 2011

3.				
(a)		$a = 0.1$	B1	
	$[F(3) = F(2) + P(Y=3) = (0.5 + 0.3)]$			
		$d = 0.8$	B1	
	$b = F(2) - a = 0.5 - 0.1$ or $a + b = 0.5$		M1	
		$b = 0.4$	A1	
	$c = 1 - F(3)$ or $1 - (a + b + 0.3)$ or $a + b + c = 0.7$			
		$c =$	A1	
	0.2			(5)
(b)	$P(3Y + 2 \geq 8) = P(Y \geq 2)$ or $1 - P(Y \leq 1)$		M1	
	$= b + 0.3 + c$ or $1 - a$ $= 0.9$		A1ft	
				(2)
				7

Notes

		Correct answers with no (or irrelevant) working score full marks
(a)	1st B1	for $a = 0.1$
	2nd B1	for $F(3) = 0.8$ or $d = 0.8$
	M1	for a method for b or c. E.g. sight of $a + b = 0.5$ or $a + b + c = 0.7$
		If their values satisfy one of these equations then score M1 provided their
	values	
		are genuine probabilities (i.e. $0 < p < 1$)
		This M1 may be implied by a correct answer for b or c
	1st A1	for b or $P(2) = 0.4$
	2nd A1	for c or $P(3) = 0.2$
(b)	M1	for rearranging to $P(Y \geq 2)$ or $1 - P(Y \leq 1)$ or selecting cases $Y = 2$, 3 and 4
	A1ft	for $0.3 +$ their $b +$ their c or $1 -$ their a, provided final answer < 1 and their
		values are probabilities.

8.

(a) $1 = p + (0.25 + 0.25 + 0.2 + 0.2)$, $\Rightarrow p = \frac{1}{10}$ or 0.1 — M1, A1

(2)

(b) $E(S) = \frac{1}{4} + 2 \times \frac{1}{4} + 4 \times \frac{1}{5} + 5 \times \frac{1}{5}$, (or equiv. in decimals) $= 2.55$ — M1, A1

(2)

(c) $E(S^2) = \frac{1}{4} + \frac{2^2}{4} + \frac{4^2}{5} + \frac{5^2}{5}$ or $0.25 + 1 + 3.2 + 5 = 9.45$ (*) — M1, A1cso

(2)

(d) $Var(S) = 9.45 - (E(S))^2$, $= 2.9475$ or $\frac{1179}{400}$ (accept awrt 2.95) — M1, A1

(2)

(e) $P(5 \text{ and } 5) = \left(\frac{1}{5}\right)^2$, $= \frac{1}{25}$ or 0.04 — M1, A1

(2)

(f) $P(4, 4, 2) = \left(\frac{1}{5}\right)^2 \times \frac{1}{4} \times 3$ $(= 0.03 \text{ or } \frac{3}{100})$ — M1, M1

$P(4, 4, 4) = \left(\frac{1}{5}\right)^3$ $(= 0.008 \text{ or } \frac{1}{125})$ — B1

P(Tom wins in 3 spins) = 0.038 — A1

(4)

(g) $P\left(\bar{5} \cap 5 \cap 5\right) + P(5 \cap \bar{5} \cap 5) = \frac{4}{5} \times \left(\frac{1}{5}\right)^2 \times 2 = 0.064$ or $\frac{8}{125}$ — M1, M1, A1

(3)

17

Notes
(a)	M1 for clear attempt to use sum of probabilities = 1 (fractions or decimals) Ans only 2/2

(b) M1 for at least 2 correct terms $(\neq 0)$ of the expression. 2.55 with no working scores M1A1
Any division by k (usually 5) in (b) or (c) or (d) scores M0

(c) M1 for at least 3 correct, non-zero terms of the expression seen, allow decimals.
A1cso for the full expression (with 9.45) seen. Must be cso but can ignore wrong p.

(d) M1 for a correct expression (9.45 seen), can ft their E(S). May see $\sum (x - "2.55")^2 \times P(X = x)$
A1 accept awrt 2.95 Answer only can score M1 for correct ft and A1 for awrt 2.95
Answer only in (e) and (f) is full marks, in (g) is no marks

(e) M1 for $\left(\frac{1}{5}\right)^2$ Condone $P(5) \times P(5) = 0.25 \times 0.25$. [Beware 0.4 is A0]

(f) 1st M1 for $\left(\frac{1}{5}\right)^2 \times \frac{1}{4}$ or 0.01 seen

2nd M1 for multiplying a $p^2 q$ probability by 3 $(p, q \in (0,1))$. B1 for $(0.2)^3$ or better seen

(g) 1st M1 for $\frac{4}{5} \times \left(\frac{1}{5}\right)^2$ or all cases considered and correct attempt at probabilities.

2nd M1 for multiplying a $p^2(1 - p)$ probability by 2. Beware $(0.4)^3 = 0.064$ is M0M0A0

6.		
(a)	$k + 2k + 3k + 4k = 1$ or $10k = 1$ $k = 0.1$ (*) [allow verification with a comment e.g. "so $k = 0.1$"]	B1cso (1)
(b)	$E(X) = 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4 = 3$	M1 A1 (2)
(c)	$E(X^2) = 1 \times 0.1 + 4 \times 0.2 + 9 \times 0.3 + 16 \times 0.4 = 10$	M1 A1 (2)
(d)	$Var(X) = 10 - 9 (= 1)$ $Var(2 - 5X) = 5^2 Var(X) = 25$	M1 M1 A1 (3)
(e)	$P(1,3) + P(2,2) = 2 \times 0.1 \times 0.3 + 0.2 \times 0.2 = 0.1$ (*)	M1 A1cso (2)
(f)	<table><tr><td>$X_1 + X_2$</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>p</td><td>0.01</td><td>0.04</td><td>0.1</td><td>**0.2**</td><td>0.25</td><td>0.24</td><td>**0.16**</td></tr></table>	B1 B1 (2)
(g)	$P(2) + P(3) = 0.05$	M1A1 (2) [14]

Q3	(a)	$2a + \frac{2}{5} + \frac{1}{10} = 1$ (or equivalent)	M1
		$a = \dfrac{1}{4}$ or 0.25	A1 (2)
	(b)	$E(X) = \underline{1}$	B1 (1)
	(c)	$E(X^2) = 1 \times \frac{1}{5} + 1 \times \frac{1}{10} + 4 \times \frac{1}{4} + 9 \times \frac{1}{5}$ (= 3.1)	M1
		$Var(X) = 3.1 - 1^2,$ $= \underline{2.1}$ or $\dfrac{21}{10}$ oe	M1 A1 (3)
	(d)	$Var(Y) = (-2)^2 Var(X),$ $= \underline{8.4}$ or $\dfrac{42}{5}$ oe	M1 A1 (2)
	(e)	$X \geq Y$ when $X = 3$ or 2, so probability $= "\frac{1}{4}" + \frac{1}{5}$	M1 A1ft
		$= \underline{\frac{9}{20}}$ oe	A1 (3)
			Total 11

	(a)	M1for a clear attempt to use $\sum P(X = x) = 1$ Correct answer only 2/2. **NB Division by 5 in parts (b), (c) and (d) seen scores 0. Do not apply ISW.**
	(b)	B1 for 1
	(c)	1st M1 for attempting $\sum x^2 P(X = x)$ at least two terms correct. Can follow through. 2nd M1 for attempting $E(X^2) - [E(X)]^2$ or allow subtracting 1 from their attempt at $E(X^2)$ provided no incorrect formula seen. Correct answer only 3/3.
	(d)	M1 for $(-2)^2 Var(X)$ or $4Var(X)$ Condone missing brackets provided final answer correct for their $Var(X)$. Correct answer only 2/2.
	(e)	Allow M1 for distribution of $Y = 6 - 2X$ and correct attempt at $E(Y^2) - [E(Y)]^2$ M1 for identifying $X = 2, 3$ 1st A1ft for attempting to find their $P(X=2) + P(X = 3)$ 2nd A1 for $\frac{9}{20}$ or 0.45

Q6	(a)			B1	

0	1	2	3
$3a$	$2a$	a	b

(1)

(b)

$3a + 2a + a + b = 1$ or equivalent, using Sum of probabilities =1   M1

$2a + 2a + 3b = 1.6$ or equivalent, using E(X)=1.6   M1

$14a = 1.4$ Attempt to solve   M1dep

$a = 0.1$ cao   B1

$b = 0.4$ cao   B1

(5)

(c)

$P(0.5 < x < 3) = P(1) + P(2)$ $3a$ or their $2a$+their a M1

$= 0.2 + 0.1$

$= 0.3$ Require $0 < 3a < 1$ to award follow through   A1 ft

(2)

(d)

$E(3X - 2) = 3E(X) - 2$ M1

$= 3 \times 1.6 - 2$

$= 2.8$ cao   A1

(2)

(e)

$E(X^2) = 1 \times 0.2 + 4 \times 0.1 + 9 \times 0.4 (= 4.2)$ M1

Var $(X) = $ "4.2" $- 1.6^2$ M1

$= 1.64$ **given answer**   cso   A1

(3)

(f)

Var$(3X - 2) = 9$ Var (X) M1

$= 14.76$ awrt 14.8   A1

(2)

[15]

Notes

6(a) Condone a clearly stated in text but not put in table.

6(b) Must be attempting to solve 2 different equations so third M dependent upon first two Ms being awarded.

Correct answers seen with no working B1B1 only, 2/5

Correctly verified values can be awarded M1 for correctly verifying sum of probabilities =1, M1 for using E(X)=1.6 M0 as no attempt to solve and B1B1 if answers correct.

6(d) 2.8 only award M1A1

6(e) Award first M for at least two non-zero terms correct. Allow first M for correct expression with a and b e.g. $E(X^2) = 6a + 9b$

Given answer so award final A1 for correct solution.

6(f) 14.76 only award M1A1

3 (a)	$\dfrac{5}{21}+\dfrac{2k}{21}+\dfrac{7}{21}+\dfrac{k}{21}=1$	M1	
	$\dfrac{12+3k}{21}=1$		
	$k=3$ * AG required for both methods	A1	(2)
(b)	$\dfrac{11}{21}$	B1	(1)
(c)	$E(X)=2\times\dfrac{5}{21}+3\times\dfrac{6}{21}+4\times\dfrac{7}{21}+6\times\dfrac{1}{7}$	M1	
	$=3\dfrac{11}{21}$ or $\dfrac{74}{21}$ or awrt 3.52	A1	(2)
(d)	$E(X^2)=2^2\times\dfrac{5}{21}+3^2\times\dfrac{6}{21}+4^2\times\dfrac{7}{21}+6^2\times\dfrac{1}{7}$	M1	
	$=14$	A1	(2)
(e)	$Var(X)=14-\left(3\dfrac{11}{21}\right)^2$	M1	
	$=1\dfrac{257}{441}$ or $\dfrac{698}{441}$ or awrt 1.6	A1	
	$Var(7X-5)=7^2\,Var(X)$	M1	
	$=77\dfrac{5}{9}$ or $\dfrac{698}{9}$ or awrt 77.6	A1	(4)
			Total 11

NOTES	
(a)	M1 Award for verification. Sub in k=3 and show $\sum x\,P(X=x)=1$. Require at least three correct terms seen or line 2 of scheme.
	A1 Correct solution only including verification.
(b)	B1 Award for exact equivalent.
(c)	M1 At least two correct terms required for method, follow through 'their k' for method. Correct answer only, award M1 A1.
(d)	M1 At least two correct terms required for method. M0 if probability is squared. Correct answer only, award M1 A1. Accept exact equivalent of 14 for A1.
(e)	M1 for use of correct formula in both. 1.6 can be implied by correct final answer. Working needs to be clearly labelled to award first method mark without second stage of calculation.
	If a new table for values of 7X – 5 is used, so Y = 7X – 5
	$E(Y^2)=\dfrac{9751}{21}$; $Var(Y)=77\dfrac{5}{9}$ or $\dfrac{698}{9}$ or awrt 77.6 Award M1A1; M1A1
	If any attempt to divide by 4 seen as part of working award M0 for that part.

3. (a)		N.B. Part (a) doesn't have to be in a table, could be a list $P(X=1) = \ldots$etc							B1, B1, B1

x	1	2	3	4	5	6
$P(X=x)$	$\dfrac{1}{36}$	$\dfrac{3}{36}$	$\dfrac{5}{36}$	$\dfrac{7}{36}$	$\dfrac{9}{36}$	$\dfrac{11}{36}$

0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306   (Accept awrt 3 s.f)   (3)

(b) $P(3) + P(4) + P(5) =, \dfrac{21}{36}$ or $\dfrac{7}{12}$ or awrt 0.583 — M1, A1 (2)

(c) $E(X) = \dfrac{1}{36} + 2\times\dfrac{3}{36} + \ldots, = \dfrac{161}{36}$ or $4.47\dot{2}$ or $4\frac{17}{36}$ — M1, A1 (2)

(d) $E(X^2) = \dfrac{1}{36} + 2^2\times\dfrac{3}{36} + \ldots, = \dfrac{791}{36}$ or full expression or $21\frac{35}{36}$ or awrt 21.97 — M1, A1

$Var(X) = \dfrac{791}{36} - \left(\dfrac{161}{36}\right)^2$, $= \underline{\mathbf{1.9714\ldots}}$ * — M1, A1c.s.o. (4)

(e) $Var(2-3X) = 9\times1.97$ or $(-3)^2\times1.97$, $= 17.73$ awrt $\underline{\mathbf{17.7}}$ or $\frac{2555}{144}$ — M1, A1 (2)

13 marks

(a) 1st B1 for $x = 1, \ldots$ 6 and at least one correct probability   N.B. $\frac{3}{36} = \frac{1}{12}$ and $\frac{9}{36} = \frac{1}{4}$

2nd B1 for at least 3 correct probabilities

3rd B1 for a fully correct probability distribution.

(b) M1   for attempt to add the correct three probabilities, ft their probability distribution

(c) M1   for a correct attempt at $E(X)$. Minimum is as printed. Exact answer only scores M1A1.

[Division by 6 at any point scores M0, no ISW. Non-exact answers with no working score M0.]

(d) 1st M1 for a correct attempt at $E(X^2)$. Minimum as printed. $\dfrac{791}{36}$ or awrt 21.97 scores M1A1.

2nd M1 for their $E(X^2) - \left(\text{their } E(X)\right)^2$.

2nd A1 cso needs awrt 1.97 <u>and</u> $\dfrac{791}{36} - \left(\dfrac{161}{36}\right)^2$ or $\dfrac{2555}{1296}$ or any fully correct expression seen.

Can accept <u>at least 4 sf</u> for both. i.e. 21.97 for $\dfrac{791}{36}$, 4.472 for $\dfrac{161}{36}$, 20.00 for $\left(\dfrac{161}{36}\right)^2$.

(e) M1   for correct use of $Var(aX + b)$ formula or a <u>full</u> method.

NB $-3^2\times1.97$ followed by awrt 17.7 scores M1A1 <u>BUT</u> $-3^2\times1.97$ alone, or followed by
-17.7, scores M0A0.