IYGB - FP1 PAPER N - QUESTION 1

EVALUATING THE DETERMINANT OF THE MATRIX AFTER SIMPLIFICATION WITH ELEMENTARY OPERATIONS

$$|\underline{C}| = \begin{vmatrix} 2 & -2 & 4 \\ 5 & x-2 & 2 \\ -1 & 3 & x \end{vmatrix} \underset{C_{13}(-2)}{\overset{C_{12}(1)}{=}} \begin{vmatrix} 2 & 0 & 0 \\ 5 & x+3 & -8 \\ -1 & 2 & x+2 \end{vmatrix}$$

EXPANDING BY THE FIRST ROW

$$\ldots = 2\begin{vmatrix} x+3 & -8 \\ 2 & x+2 \end{vmatrix} = 2\left[(x+2)(x+3)+16\right]$$

$$= 2\left[x^2+5x+6+16\right]$$

$$= 2\left[x^2+5x+22\right]$$

$$= 2\left[\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+22\right]$$

$$= 2\left(x+\frac{5}{2}\right)^2+\frac{63}{2} > 0 \quad \text{FOR ALL } x$$

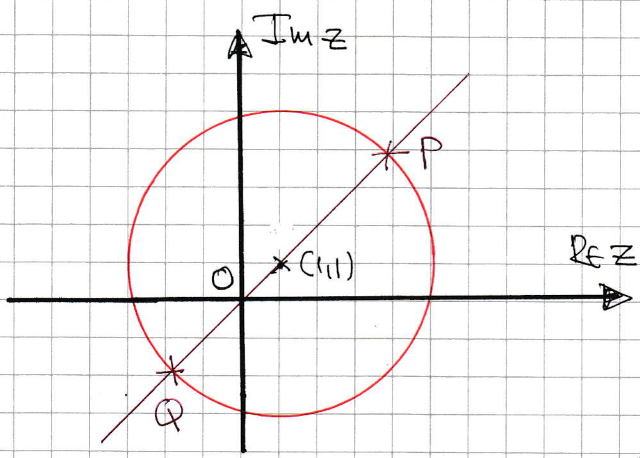
THEREFORE C IS NON SINGULAR FOR ALL x

IYGB - FP1 PAPER N - QUESTION 2

a) THIS IS A STANDARD CIRCLE LOCUS

$$|z-1-i|=4$$
$$|z-(1+i)|=4$$

i.e. CENTRE AT $(1,1)$, RADIUS 4



b) LOOKING AT THE DIAGRAM ABOVE & NOTING THAT $|z|$ REPRESENTS THE DISTANCE OF A POINT FROM THE ORIGIN

- DISTANCE OF THE CENTRE FROM O IS $\sqrt{2}$
- $|z|_{MAX} = \text{RADIUS} + \sqrt{2} = 4+\sqrt{2}$ (POINT P)
- $|z|_{MIN} = \text{RADIUS} - \sqrt{2} = 4-\sqrt{2}$ (POINT Q)

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$$\boxed{\sum_{r=1}^{n}\left(\frac{1}{4r^2-1}\right) = \frac{n}{2n+1}}$$

- TESTING THE BASE CASE, IE $n=1$

$$\text{LHS} = \sum_{r=1}^{1} \frac{1}{4r^2-1} = \frac{1}{4\times1^3-1} = \frac{1}{3}$$

$$\text{RHS} = \frac{1}{2\times1+1} = \frac{1}{3}$$

$\therefore$ THE RESULT HOLDS FOR $n=1$

- SUPPOSE THAT THE RESULT HOLDS FOR $n=k \in \mathbb{N}$

$$\sum_{r=1}^{k}\left(\frac{1}{4r^2-1}\right) = \frac{k}{2k+1}$$

$$\sum_{r=1}^{k}\left(\frac{1}{4r^2-1}\right) + \frac{1}{4(k+1)^2-1} = \frac{k}{2k+1} + \frac{1}{4(k+1)^2-1}$$

$$\sum_{r=1}^{k+1}\left(\frac{1}{4r^2-1}\right) = \frac{k}{2k+1} + \frac{1}{[2(k+1)+1][2(k+1)-1]}$$

$$\sum_{r=1}^{k+1}\left(\frac{1}{4r^2-1}\right) = \frac{k}{2k+1} + \frac{1}{(2k+3)(2k+1)}$$

$$\sum_{r=1}^{k+1}\left(\frac{1}{4r^2-1}\right) = \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$

$$\sum_{r=1}^{k+1}\left(\frac{1}{4r^2-1}\right) = \frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+1}$$

- IF THE RESULT HOLDS FOR $n=k \in \mathbb{N}$, THEN IT ALSO HOLDS FOR $n=k+1$

SINCE THE RESULT HOLDS FOR $n=1$, THEN IT MUST HOLD $\forall n \in \mathbb{N}$

IYGB - FP1 PAPER N - QUESTION 4

$$\boxed{z = 3+i \qquad w = 1+2i}$$

- FIRST FIND $\frac{z}{w}$.

$$\frac{z}{w} = \frac{3+i}{1+2i} = \frac{(3+i)(1-2i)}{(1+2i)(1-2i)} = \frac{3-6i+i-2i^2}{1^2+2^2} = \frac{3-5i+2}{5}$$

$$= \frac{5-5i}{5} = 1-i$$

- HENCE WE NOW HAVE

$$\Longrightarrow \left|\frac{z}{w}+\lambda\right| = \sqrt{\lambda+2}$$

$$\Longrightarrow |(1-i)+\lambda| = \sqrt{\lambda+2}$$

$$\Longrightarrow |(1+\lambda)-i| = \sqrt{\lambda+2}$$

$$\Longrightarrow \sqrt{(\lambda+1)^2+1^2} = \sqrt{\lambda+2}$$

$$\Longrightarrow (\lambda+1)^2+1 = \lambda+2$$

$$\Longrightarrow \lambda^2+2\lambda+1+1 = \lambda+2$$

$$\Longrightarrow \lambda^2+\lambda = 0$$

$$\Longrightarrow \lambda(\lambda+1) = 0$$

$$\Longrightarrow \lambda = \begin{cases} 0 \\ -1 \end{cases}$$

IYGB - FP1 PAPER N - QUESTION 5

a) BY "MULTIPLICATION"

$$\begin{pmatrix} 2 & a \\ 3 & b \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \implies \begin{pmatrix} 2\times1 + a\times1 \\ 3\times1 + b\times1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\implies \begin{array}{l} a+2=1 \\ b+3=3 \end{array}$$

$$\therefore \ a=-1 \ \ \& \ \ b=0$$

b) VERIFY BY CALCULATING EACH SIDE SEPARATELY

- $$\underline{A}^2 = \underline{A}\underline{A} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2\times2-1\times3 & 2(-1)+0 \\ 3\times2+0 & 3(-1)+0 \end{pmatrix}$$

$$\therefore \ \underline{A}^2 = \begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}$$

- $$2\underline{A} - 3\underline{I} = 2\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 6 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}$$

$$\therefore \ \underline{A}^2 = 2\underline{A} - 3\underline{I}$$

c) I)

$$\underline{A}^2 = 2\underline{A} - 3\underline{I}$$

$$\implies \underline{A}^2\underline{A} = 2\underline{A}\underline{A} - 3\underline{I}\underline{A}$$

$$\implies \underline{A}^3 = 2\underline{A}^2 - 3\underline{A}$$

$$\implies \underline{A}^3 = 2(2\underline{A} - 3\underline{I}) - 3\underline{A}$$

$$\implies \underline{A}^3 = \underline{A} - 6\underline{I}$$

AS REQUIRED

II)

$$\underline{A}^2 = 2\underline{A} - 3\underline{I}$$

$$\implies \underline{A}\underline{A}\underline{A}^{-1} = 2\underline{A}\underline{A}^{-1} - 3\underline{I}\underline{A}^{-1}$$

$$\implies \underline{A}\underline{I} = 2\underline{I} - 3\underline{A}^{-1}$$

$$\implies \underline{A} = 2\underline{I} - 3\underline{A}^{-1}$$

$$\implies 3\underline{A}^{-1} = 2\underline{I} - \underline{A}$$

$$\implies \underline{A}^{-1} = \frac{1}{3}(2\underline{I} - \underline{A})$$

IYGB - FP1 PAPER N - QUESTION 6

START BY MANIPULATING THE FUNCTION

$$y = \frac{4+\sin x\cos x}{\cos 2x} = \frac{4+\frac{1}{2}(2\sin x\cos x)}{\cos 2x}$$

$$= \frac{4+\frac{1}{2}\sin 2x}{\cos 2x} = 4\sec 2x + \frac{1}{2}\tan 2x$$

HENCE THE VOLUME OF REVOLUTION IS GIVEN BY

$$\Longrightarrow V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \left(4\sec 2x + \frac{1}{2}\tan 2x\right)^2 \, dx$$

$$\Longrightarrow V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} 16\sec^2 2x + 4\sec 2x\tan 2x + \frac{1}{4}\tan^2 2x \, dx$$

$$\Longrightarrow V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} 16\sec^2 2x + 4\sec 2x\tan 2x + \frac{1}{4}(\sec^2 2x - 1) \, dx$$

$$\Longrightarrow V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{65}{4}\sec^2 2x + 4\sec 2x\tan 2x - \frac{1}{4} \, dx$$

$$\Longrightarrow V = \pi \left[\frac{65}{8}\tan 2x + 2\sec 2x - \frac{1}{4}x\right]_{\frac{\pi}{12}}^{\frac{\pi}{6}}$$

$$\Longrightarrow V = \pi \left[\left(\frac{65}{8}\sqrt{3} + 4 - \frac{\pi}{24}\right) - \left(\frac{65}{24}\sqrt{3} + \frac{4\sqrt{3}}{3} - \frac{\pi}{48}\right)\right]$$

$$\Longrightarrow V = \pi \left[4 + \frac{49}{12}\sqrt{3} - \frac{\pi}{48}\right] \approx 34.6$$

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- USING A SUBSTITUTION HERE

$$y = \frac{4}{3}(x-1)$$
$$3y = 4x - 4$$
$$4x = 3y + 4$$

- MANIPULATE THE CUBIC FOR SIMPLICITY

$\Rightarrow 16x^3 - 8x^2 + 4x - 1 = 0$

$\Rightarrow 64x^3 - 32x^2 + 16x - 4 = 0$ ×4

$\Rightarrow (4x)^3 - 2(4x)^2 + 4(4x) - 4 = 0$

$\Rightarrow (3y+4)^3 - 2(3y+4)^2 + 4(3y+4) - 4 = 0$

NOW:

$$\boxed{\begin{aligned}(A+B)^3 &= A^3 + 3A^2B + 3AB^2 + B^3\\ (3y+4)^3 &= 27y^3 + 3(3y)^2 \times 4 + 3(3y) \times 16 + 64\\ &= 27y^3 + 108y^2 + 144y + 64\end{aligned}}$$

$\Rightarrow 27y^3 + 108y^2 + 144y + 64 - 2(9y^2 + 24y + 16) + 12y + 16 - 4 = 0$

$$\Rightarrow \left.\begin{array}{r} 27y^3 + 108y^2 + 144y + 64 \\ - 18y^2 - 48y - 32 \\ 12y + 12 \end{array}\right\} = 0$$

$\Rightarrow 27y^3 + 90y^2 + 108y + 44 = 0$

IYGB - FP1 PAPER N - QUESTION 8

EXPAND THE SUMMATION SO WE CAN USE STANDARD RESULTS

$$\sum_{r=0}^{n}\left[2r(2r^2-3r-1)+(n+1)\right]$$

$$=\sum_{r=0}^{n}\left[4r^3-6r^2-2r+(n+1)\right]$$

$$=4\sum_{r=0}^{n}r^3-6\sum_{r=0}^{n}r^2-2\sum_{r=0}^{n}r+\sum_{r=0}^{n}(n+1)$$

THE SUMMAND HAS NO DEPENDENCE ON r, SO IT MAY BE FACTORIZED OUT

NOTE THE THE FIRST TERM IN THE FIRST 3 SUMMATIONS IS ZERO, SO WE MAY START THESE SUMMATIONS FROM $r=1$

$$=4\sum_{r=1}^{n}r^3-6\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r+(n+1)\sum_{r=0}^{n}1$$

USING STANDARD RESULTS

$$=4\times\frac{1}{4}n^2(n+1)^2-6\times\frac{1}{6}n(n+1)(2n+1)-2\times\frac{1}{2}n(n+1)+(n+1)\times(n+1)$$

FROM $r=0$ TO $r=n$, THERE ARE $n+1$ TERMS

$$=n^2(n+1)^2-n(n+1)(2n+1)-n(n+1)+(n+1)^2$$

$$=n(n+1)\left[n(n+1)-(2n+1)-1\right]+(n+1)^2$$

$$=n(n+1)\left[n^2-n-2\right]+(n+1)^2$$

$$=n(n+1)(n+1)(n-2)+(n+1)^2$$

$$=(n+1)^2\left[n(n-2)+1\right]$$

$$=(n+1)^2(n^2-2n+1)$$

$$=(n+1)^2(n-1)^2=\left[(n+1)(n-1)\right]^2$$

$$=(n^2-1)^2$$

AS REQUIRED

IYGB - FP1 PAPER N - QUESTION 9

a) DOTTING THE DIRECTION VECTORS

$\Longrightarrow (1,1,p)\cdot(1,1,1)=0$

$\Longrightarrow 1+1+p=0$

$\Longrightarrow p=-2$

b)

$$\boxed{\begin{aligned} r_1 &= (8,d,-3)+\lambda(1,1,-2) = (\lambda+8,\ \lambda+d,\ -3-2\lambda) \\ r_2 &= (3,-4,-5)+\mu(1,1,1) = (\mu+3,\ \mu-4,\ \mu-5) \end{aligned}}$$

EQUATE i & j

$$\left.\begin{aligned} i:&\quad \lambda+8=\mu+3 \\ j:&\quad -3-2\lambda=\mu-5 \end{aligned}\right\}\ \text{SUBTRACT} \Longrightarrow 3\lambda+11=8$$

$\Longrightarrow 3\lambda=-3$

$\Longrightarrow \lambda=-1$ & $\mu=4$

EQUATE k

$\lambda+d=\mu-4$

$-1+d=4-4$

$d=1$

& USING $\mu=4$

$D(7,0,-1)$

c) NEED THE DIRECTION OF l_3 WHICH IS PERPENDICULAR TO l_1 & l_2

$$\left.\begin{aligned} (x,y,z)\cdot(1,1,1)&=0 \\ (x,y,z)\cdot(1,1,-2)&=0 \end{aligned}\right\} \Longrightarrow \begin{aligned} x+y+z&=0 \\ x+y-2z&=0 \end{aligned}$$

LET $x=1$ $\Longrightarrow \begin{cases} y+z=-1 \\ y-2z=-1 \end{cases}$

$\Longrightarrow z=0$

&

$y=-1$

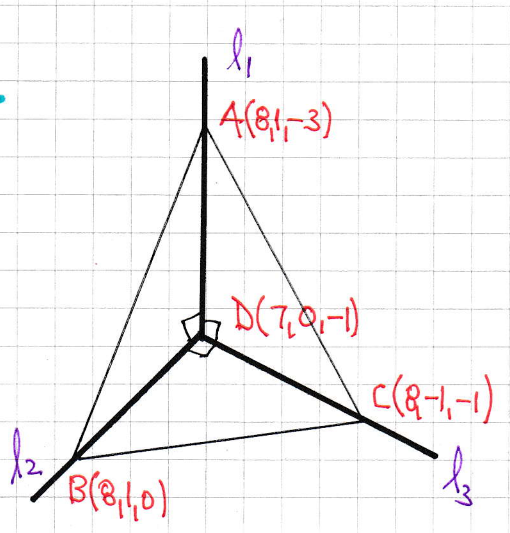
IYGB - FP1 PAPER N - QUESTION 9

USING THE MUTUAL PERPENDICULAR DIRECTION $(1,-1,0)$ AND THE POINT $D(7,0,-1)$

$$\therefore \ l_3 = (7,0,-1) + t(1,-1,0)$$

$$l_3 = (t+7, -t, -1)$$

d) CALCULATING THE RELEVANT LENGTHS



$$|AD| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$$

$$|BD| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$|DC| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$$

AREA OF A PYRAMID $= \frac{1}{3}(\text{BASE AREA}) \times \text{HEIGHT}$

$$= \frac{1}{3}\left(\frac{1}{2}|BD||DC|\right) \times |AD|$$

$$= \frac{1}{6} \times \sqrt{3} \times \sqrt{2} \times \sqrt{6}$$

$$= \frac{1}{6} \times 6$$

$$= 1 \ \text{UNIT}^3$$