IYGB - MP1 PAPER Z - QUESTION 1

USING STANDARD REARRANGING TECHNIQUES

$$\frac{\sqrt{p}}{2p+\sqrt{p}} = \frac{2\sqrt{p}-q}{3p+q}$$

$$\Rightarrow \sqrt{p}(3p+q) = (2p+\sqrt{p})(2\sqrt{p}-q)$$

$$\Rightarrow 3p\sqrt{p} + q\sqrt{p} = 4p\sqrt{p} - 2pq + 2p - q\sqrt{p}$$

$$\Rightarrow 2q\sqrt{p} + 2pq = p\sqrt{p} + 2p$$

$$\Rightarrow q\left[2\sqrt{p} + 2p\right] = p\sqrt{p} + 2p$$

$$\Rightarrow q = \frac{p\sqrt{p} + 2p}{2\sqrt{p} + 2p}$$

$$\Rightarrow q = \frac{p\cancel{\sqrt{p}} + 2\sqrt{p}\cancel{\sqrt{p}}}{2\cancel{\sqrt{p}} + 2\sqrt{p}\cancel{\sqrt{p}}} = \frac{p+2\sqrt{p}}{2+2\sqrt{p}}$$

// AS REQUIRED

ALTERNATIVE

$$\frac{\sqrt{p}}{2p+\sqrt{p}} = \frac{2\sqrt{p}-q}{3p+q}$$

$$\Rightarrow \frac{1}{2\sqrt{p}+1} = \frac{2\sqrt{p}-q}{3p+q}$$

DIVIDE "TOP & BOTTOM" OF THE FRACTION IN THE L.H.S BY $\sqrt{p}$

$$\Rightarrow 3p+q = (2\sqrt{p}-q)(2\sqrt{p}+1)$$

$$\Rightarrow 3p+q = 4p + 2\sqrt{p} \quad 2q\sqrt{p} - q$$

$$\Rightarrow 2q + 2q\sqrt{p} = p + 2\sqrt{p}$$

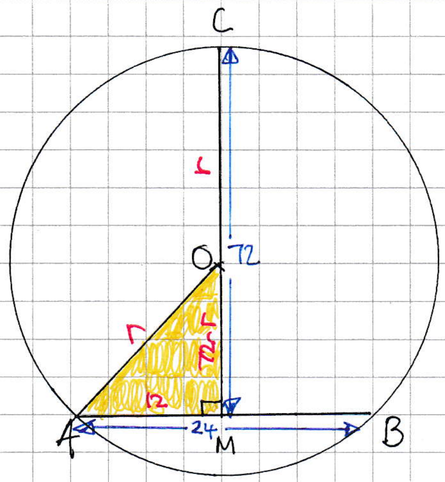
$$\Rightarrow q(2+2\sqrt{p}) = p + 2\sqrt{p}$$

$$\Rightarrow q = \frac{p+2\sqrt{p}}{2+2\sqrt{p}}$$

// AS BEFORE

IYGB - MP1 PAPER Z - QUESTION 2

LOOKING AT THE DIAGRAM ON THE TRIANGLE AMO



BY PYTHAGORAS

$$\Rightarrow |AM|^2 + |MO|^2 = |OA|^2$$

$$\Rightarrow 12^2 + (72-r)^2 = r^2$$

$$\Rightarrow 144 + 5184 - 144r + \cancel{r^2} = \cancel{r^2}$$

$$\Rightarrow 5328 = 144r$$

$$\Rightarrow r = 37$$

IYGB - MP1 PAPER Z - QUESTION 3

EXPAND THE EXPRESSION UP TO x^2, IN TERMS OF a & b

$$(2+ax)^2(1+bx)^6 = (4+4ax+a^2x^2)\left[1+\frac{6}{1}(bx)^1+\frac{6\times5}{1\times2}(bx)^2+\dots\right]$$

$$= (4+4ax+a^2x^2)(1+6bx+15b^2x^2+\dots)$$

$$= 4 + 24bx + 60b^2x^2 + \dots$$
$$\quad\; 4ax + 24abx^2 + \dots$$
$$\quad\quad a^2x^2 + \dots$$

$$= 4 + (4a+24b)x + (60b^2+24ab+a^2)x^2 + \dots$$

COMPARING COEFFICIENTS

- $4a+24b = 44$

$$a+6b = 11$$
$$a = 11-6b \longrightarrow$$

- $60b^2+24ab+a^2 = 85$

$$60b^2+24b(11-6b)+(11-6b)^2 = 85$$
$$60b^2+264b-144b^2+121-132b+36b^2 = 85$$
$$-48b^2+132b+36 = 0$$
$$48b^2-132b-36 = 0$$
$$4b^2-11b-3 = 0$$
$$(4b+1)(b-3) = 0$$
$$b = \begin{cases} 3 \\ -\frac{1}{4} \end{cases}$$

$$a = \begin{cases} 11-6\times3 = -7 \\ 11-6\left(-\frac{1}{4}\right) = \frac{25}{2} \end{cases}$$

EITHER $a=-7$ & $b=3$

OR $a=\frac{25}{2}$ & $b=-\frac{1}{4}$

IYGB - MP1 PAPER Z - QUESTION 4

REARRANGE THE EXPRESSION AS A QUADRATIC IN x

$$f(x) = x^2 + 2x - m(x^2 - 2x + 2) - 2$$

$$f(x) = x^2 + 2x - mx^2 + 2mx - 2m - 2$$

$$f(x) = \underbrace{(1-m)}_{a}x^2 + \underbrace{(2+2m)}_{b}x + \underbrace{(-2m-2)}_{c}$$

FOR DISTINCT REAL ROOTS OF $f(x)=0$, $b^2 - 4ac > 0$

$$\Rightarrow (2+2m)^2 - 4(1-m)(-2m-2) > 0$$

$$\Rightarrow 4(1+m)^2 + 4(1-m)(2m+2) > 0$$

$$\Rightarrow (1+m)^2 + (1-m)(2m+2) > 0$$

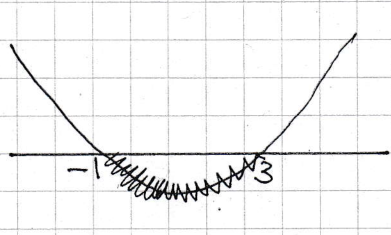
$$\Rightarrow 1 + 2m + m^2 + \cancel{2m} + 2 - 2m^2 - \cancel{2m} > 0$$

$$\Rightarrow -m^2 + 2m + 3 > 0$$

$$\Rightarrow m^2 - 2m - 3 < 0$$

$$\Rightarrow (m-3)(m+1) < 0$$

CRITICAL VALUES $\left\langle \begin{array}{l} -1 \\ 3 \end{array} \right.$



$-1 < m < 3$

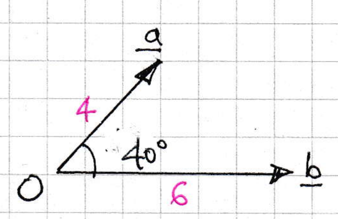
$m \neq 1$

OR $\{-1 < m < 1\} \cup \{1 < m < 3\}$

IYGB – MP1 PAPER Z – QUESTION 5

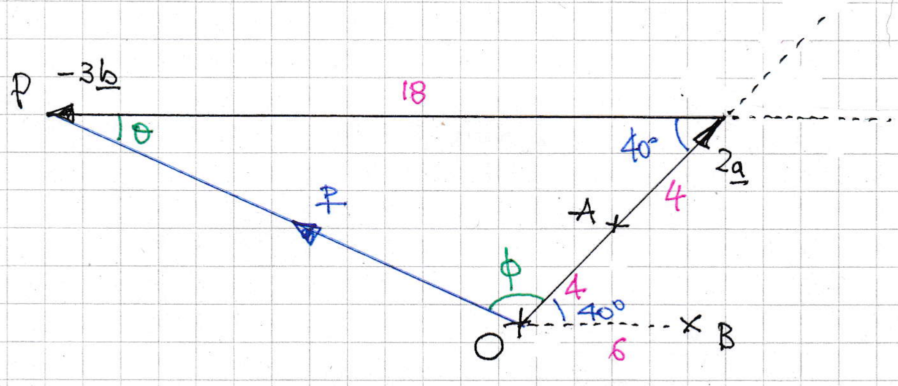
$|\underline{a}| = 4 \qquad |\underline{b}| = 6 \qquad A\hat{O}B = 40°$

START WITH A DIAGRAM



REDRAW SHOWING THE POSITION OF P

$\overrightarrow{OP} = 2\underline{a} - 3\underline{b}$



BY THE COSINE RULE

$|\underline{p}|^2 = 18^2 + 8^2 - 2 \times 18 \times 8 \times \cos 40$

$|\underline{p}|^2 = 324 + 64 - 288\cos 40°$

$|\underline{p}|^2 = 167.379...$

$|\underline{p}| = 12.93751...$

$\therefore |OP| = 12.94$

BY THE SINE RULE.

$\frac{|p|}{\sin 40°} = \frac{8}{\sin\theta}$

$\sin\theta = \frac{8\sin 40}{12.93751...}$

$\theta \approx 23.42°...$

$\therefore$ REQUIRED ANGLE ϕ

$180 - (40 + 23.42°)$

$\simeq 117°$

IYGB – MP1 PAPER Z – QUESTION 6

a) $f(x) = 2\log_4 x$

$g(x) = 1 + 2\log_4 x = 1 + f(x) = f(x) + 1$

$\therefore$ TRANSLATION BY $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

b) $g(x) = 1 + 2\log_4 x$

$g(x) = \log_4 4 + \log_4 x^2$

$g(x) = \log_4 (4x^2)$

$g(x) = \log_4 \left[(2x)^2\right]$

$g(x) = 2\log_4 (2x)$

$g(x) = f(2x)$

OR

$g(x) = \log_4 4 + 2\log_4 x$

$g(x) = \log_4 2^2 + 2\log_4 x$

$g(x) = 2\log_4 2 + 2\log_4 x$

$g(x) = 2\left[\log_4 2 + \log_4 x\right]$

$g(x) = 2\log(2x)$

$\therefore$ HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

(STRETCH PARALLEL TO THE x AXIS, BY S.F $\frac{1}{2}$)

IYGB - MP1 PAPER Z - QUESTION 7

THE EQUATION OF l IS GIVEN BY

$$y - y_0 = m(x - x_0)$$
$$y - 5 = 3(x - 4)$$
$$y - 5 = 3x - 12$$
$$y = 3x - 7$$

LET THE REQUIRED POINT HAVE CO-ORDINATES $Q(a, 3a-7)$

$$\Rightarrow |PQ| = 3\sqrt{10}$$

$$\left\{ \begin{array}{l} P(4,5) \\ Q(a, 3a-7) \end{array} \right\}$$

$$\Rightarrow \sqrt{(x_2-x_1)^2 - (y_2-y_1)^2} = 3\sqrt{10}$$
$$\Rightarrow \sqrt{(a-4)^2 + (3a-7-5)^2} = 3\sqrt{10}$$
$$\Rightarrow \sqrt{(a-4)^2 + (3a-12)^2} = 3\sqrt{10}$$
$$\Rightarrow (a-4)^2 + (3a-12)^2 = 9 \times 10$$
$$\Rightarrow \left. \begin{array}{l} a^2 - 8a + 16 \\ 9a^2 - 72a + 144 \end{array} \right\} = 90$$
$$\Rightarrow 10a^2 - 80a + 160 = 90$$
$$\Rightarrow 10a^2 - 80a + 70 = 0$$
$$\Rightarrow a^2 - 8a + 7 = 0$$
$$\Rightarrow (a-7)(a-1) = 0$$

$$a = \left\langle \begin{array}{l} 1 \\ 7 \end{array} \right. \qquad 3a - 7 = \left\langle \begin{array}{l} -4 \\ 14 \end{array} \right.$$

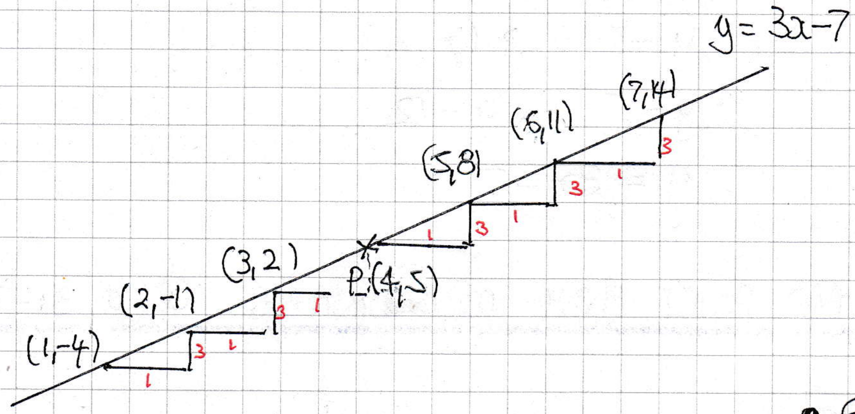
$\therefore$ $Q(1, -4)$

OR

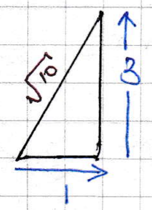
$Q(7, 14)$

IYGB - MP1 PAPER Z - QUESTION 7

ALTERNATIVE BY GEOMETRY



- GRADIENT 3



- DISTANCE $3\sqrt{10}$ is 3 "GRADIENT" STEPS FROM P
- SEE DIAGRAM FOR THE ANSWERS

$Q(1,-4)$ OR $Q(7,14)$

IYGB - MP1 PAPER Z - QUESTION 8

ALTHOUGH IT IS "TEMPTING" TO START FROM THE TWO CONDITIONS GIVEN ($t=0, h=2$ & $t=2, h=3.81$), IT BEST TO START FROM

"LIFETIME MAX HEIGHT OF 12m" $\Rightarrow$ AS $t \to \infty$ $\quad h \to 12$

$\Rightarrow$ AS $t \to \infty$ $\quad e^{-kt} \to 0$

$Be^{-kt} \to 0$

$h \to A$

$\therefore \underline{\underline{A = 12}}$

$$\boxed{h = 12 - Be^{-kt}}$$

- $t=0, h=2$

$2 = 12 - Be^{0}$

$\underline{\underline{B = 10}}$

- $t=2, h=3.81$

$3.81 = 12 - Be^{-2k}$

$3.81 = 12 - 10e^{-2k}$

$10e^{-2k} = 8.19$

$e^{-2k} = 0.819$

$-2k = \ln(0.819)$

$\underline{\underline{k = 0.099835\ldots \approx 0.1}}$

FINALLY WE HAVE

$\Rightarrow h = 12 - 10e^{-0.1t}$

$\Rightarrow 10 = 12 - 10e^{-0.1t}$

$\Rightarrow 10e^{-0.1t} = 2$

$\Rightarrow e^{-0.1t} = \frac{1}{5}$

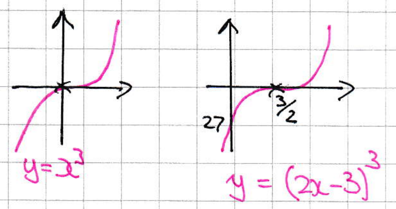
$\Rightarrow e^{0.1t} = 5$

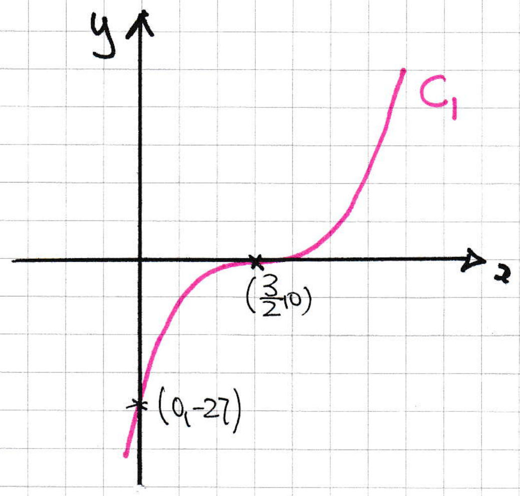
$\Rightarrow 0.1t = \ln 5$

$\therefore t = 10\ln 5 \approx 16.1$

IYGB - MP1 PAPER Z - QUESTION 9

a) • $C_1: y = (2x-3)^3$



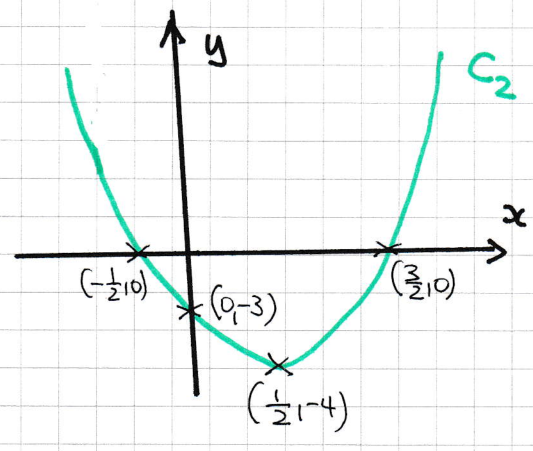


• $C_2: \quad y = (2x-1)^2 - 4$

$$y = (2x-1-2)(2x-1+2)$$

$$y = (2x-3)(2x+1)$$

$$\left(\tfrac{3}{2}, 0\right), \left(-\tfrac{1}{2}, 0\right), (0, -3)$$



b) $\Rightarrow (2x-1)^2 + (3-2x)^3 = 4$

$$\Rightarrow (2x-1)^2 - 4 = -(3-2x)^3$$

$$\Rightarrow (2x-1)^2 - 4 = +(2x-3)^3$$

USING THE FACTORIZATION FOR THE QUADRATIC FROM ABOVE

$$\Rightarrow (2x-3)(2x+1) = (2x-3)^3$$

PREFERABLY NOT EXPANDING AS THERE IS A COMMON FACTOR

$$\Rightarrow 0 = (2x-3)^3 - (2x-3)(2x+1) = 0$$

$$\Rightarrow 0 = (2x-3)\left[(2x-3)^2 - (2x+1)\right]$$

$$\Rightarrow 0 = (2x-3)\left[4x^2 - 12x + 9 - 2x - 1\right]$$

IYGB - MP1 PAPER Z - QUESTION 9

$$\Longrightarrow 0 = (2x-3)(4x^2-14x+8)$$

EITHER

$2x-3=0$

$2x=3$

$x=\frac{3}{2}$

OR

$4x^2-14x+8=0$

$2x^2-7x+4=0$

QUADRATIC FORMULA

$\Delta = (-7)^2-4\times2\times4$

$\Delta = 49-32$

$\Delta = 17$

$$x = \frac{-(-7)\pm\sqrt{17}}{2\times2}$$

$$x = \frac{7\pm\sqrt{17}}{4}$$

THUS THE THREE SOLUTIONS ARE

$$x = \begin{cases} \frac{3}{2} \\ \frac{1}{4}(7+\sqrt{17}) \\ \frac{1}{4}(7-\sqrt{17}) \end{cases}$$

IYGB - MP1 PAPER Z - QUESTION 10

a) SOLVING SIMULTANEOUSLY

$$x^2+y^2-6x=16$$
$$x^2+y^2-18x+16y=80$$

SUBTRACT

$$12x-16y=-64$$
$$16y-12x=64$$
$$4y-3x=16$$
$$3x=4y-16$$
$$9x^2=16y^2-128y+256$$

NOW MULTIPLYING THE FIRST EQUATION BY 9

$$9x^2+9y^2-54x=144$$
$$9x^2+9y^2-18(3x)=144$$
$$16y^2-128y+256+9y^2-18(4y-16)=144$$
$$16y^2-128y+256+9y^2-72y+288=144$$
$$25y^2-200y+400=0$$
$$y^2-8y+16=0$$
$$(y-4)^2=0$$

$3x=4y-16$

$y=4$ REPEATED, INDEED THEY TOUCH AT $(0,4)$

b) FIRSTLY WE NEED THE CIRCLE PARTICULARS

$$x^2+y^2-6x=16$$
$$(x-3)^2-9+y^2=16$$
$$(x-3)^2+y^2=25$$

$(3,0)$, radius 5

$$x^2+y^2-18x+16y=80$$
$$x^2-18x+y^2+16y=80$$
$$(x-9)^2-81+(y+8)^2-64=80$$
$$(x-9)^2+(y+8)^2=225$$

$(9,-8)$, radius 15

IYGB, MP1 PAPER Z – QUESTION 10

DISTANCE BETWEEN THE CENTRES $(3,0)$ & $(9,-8)$

$$d = \sqrt{(3-9)^2 + (0+8)^2}$$

$$d = \sqrt{36 + 64}$$

$$d = \sqrt{100}$$

$$d = 10$$

TOUCHING EXTERNALLY REQUIRES $d = 15 + 5 = 20$

TOUCHING INTERNALLY REQUIRES $d = 15 - 5 = 10$

$\therefore$ INTERNALLY

IYGB - MP1 PAPER Z - QUESTION 11

a) LINE: $5x+2y=q$ CUBIC: $y=x^3-4x^2+px+4$

$$\frac{dy}{dx}=3x^2-8x+p$$

WHEN $x=1$ THE y CO-ORDS ARE IDENTICAL (POINT A)

- $5+2y=q$
- $y=1-4+p+4$
  $y=p+1$

THE GRADIENT AT B, WHERE $x=2$, IS ZERO

$$\Rightarrow 0=12-16+p$$
$$\Rightarrow p=4$$

& THE y CO-ORD OF A IS 5 $(y=p+1)$

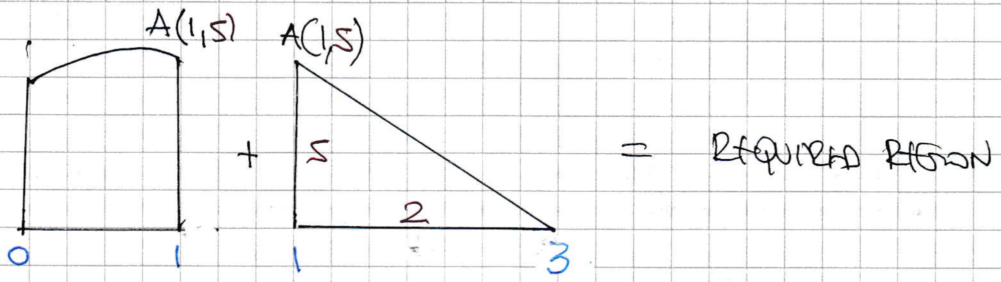
$$\Rightarrow 5+2y=q$$
$$\Rightarrow 5+10=q$$
$$\Rightarrow q=15$$

b) FIND THE x INTERCEPT OF L

$$5x+2y=15$$
$$5x+2\times0=15$$
$$x=3$$

IYGB - MP1 PAPER Z - QUESTION 11

NOW LOOKING AT THE "PICTORIAL" EQUATION



$\frac{1}{2}\times5\times2=5$

$$\int_0^1 x^3-4x^2+4x+4\ dx = \left[\tfrac{1}{4}x^4-\tfrac{4}{3}x^3+2x^2+4x\right]_0^1$$

$$= \left(\tfrac{1}{4}-\tfrac{4}{3}+2+4\right)-(0)$$

$$= \frac{59}{12}$$

THE REQUIRED REGION HAS AREA

$$\frac{59}{12}+5 = \frac{119}{12}$$

IYGB – MP1 PAPER Z – QUESTION 12

ASSERTION: IF $x \in \mathbb{R}, x>0$ THEN $x^4 + x^{-4} \geqslant 2$

PROOF BY CALCULUS

- LET $f(x) = x^4 + x^{-4} = x^4 + \frac{1}{x^4}$, $x \in \mathbb{R}, x>0$

AS $x \to +\infty$, $f(x) \to +\infty$ $\left(f(x) \sim x^4 \right)$

AS $x \to 0^+$, $f(x) \to +\infty$ $\left(f(x) \sim \frac{1}{x^4} \right)$

- LOOK FOR STATIONARY VALUES

$\Rightarrow f'(x) = 4x^3 - 4x^{-5} = 4x^{-5}(x^8 - 1)$

SOLVING FOR ZERO

$\Rightarrow \frac{4}{x^5}(x^8 - 1) = 0$

$\Rightarrow x^8 - 1 = 0$ $\left(\frac{4}{x^5} \neq 0 \right)$

$\Rightarrow x = \pm 1$ ONLY REAL SOLUTIONS

$\Rightarrow x = 1$ ONLY POSITIVE REAL SOLUTION

$\therefore f(1) = 1^4 + 1^{-4} = 2$

- AS $f(x)$ TENDS TO INFINITY AS $x \to \infty$ OR $x \to 0^+$, THEN $(1,2)$ IS MORE THAN A LOCAL MINIMUM, IT A "PROPER" MINIMUM

$\Rightarrow f(x) \geqslant 2$ WHICH IMPLIES $x^4 + x^{-4} \geqslant 2$

$x \in \mathbb{R}, x>0$

IYGB - MP1 PAPER Z - QUESTION 12

PROOF WITHOUT CALCULUS

START FROM THE FACT THAT ANY SQUARED EXPRESSION IS NON NEGATIVE

$$\Rightarrow (x^4-1)^2 \geqslant 0$$

$$\Rightarrow x^8 - 2x^4 + 1 \geqslant 0$$

$$\Rightarrow x^8 + 1 \geqslant 2x^4$$

AS $x>0$ WE MAY SAFELY DIVIDE THE INEQUALITY

$$\Rightarrow \frac{x^8+1}{x^4} \geqslant 2$$

$$\Rightarrow x^4 + x^{-4} \geqslant 2$$

AS REQUIRED