Bronze

1. A discrete random variable X has a probability function as shown in the table below, where a and b are constants.

x	0	1	2	3
$P(X = x)$	0.2	0.3	b	a

Given that $E(X) = 1.7$,

(a) find the value of a and the value of b.

(5)

Find

(b) $P(0 < X < 1.5)$,

(1)

(c) $E(2X – 3)$.

(2)

(d) Show that $Var(X) = 1.41$.

(3)

(e) Evaluate $Var(2X – 3)$.

(2)

(Total 13 marks)

Silver

2. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{n}, \qquad x = 1, 2, ..., n.$$

Given that $E(X) = 5$,

(a) show that $n = 9$.

(3)

Find

(b) P($X < 7$),

(2)

(c) Var (X).

(4)
(Total 9 marks)

Gold

3. The discrete random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} k(2 - x), & x = 0, 1, 2, \\ k(x - 2), & x = 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = 0.25$.

(2)

(b) Find E(X) and show that E(X^2) = 2.5.

(4)

(c) Find Var($3X - 2$).

(3)

Two independent observations X_1 and X_2 are made of X.

(d) Show that P($X_1 + X_2 = 5$) = 0.

(1)

(e) Find the complete probability function for $X_1 + X_2$.

(3)

(f) Find P($1.3 \leq X_1 + X_2 \leq 3.2$).

(3)
(Total 16 marks)