Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3 - 5\times(-2)^2 + a\times-2 + a = 0$	M1	3.1a
	Solves linear equation $2a - a = -36 \Rightarrow a =$	dM1	1.1b
	$\Rightarrow a = -36$	A1	1.1b
		(3 marks)	

Notes:

M1: Selects a suitable method given that $(x + 2)$ is a factor of f(x)
Accept either setting f(-2) = 0 or attempted division of f(x) by ($x + 2$)

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a = .. \Rightarrow a =$

A1: $a = -36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\dfrac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
		(3 marks)	

Notes:

(a)

B1: Accept a response of the type 'They use $\dfrac{\cos\theta}{\sin\theta} = \tan\theta$. This is incorrect as $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta = \dfrac{1}{2}$ not $\tan\theta = 2$'

Accept also statements such as 'it should be $\cot\theta = 2$'

(b)
B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta = 2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°) = +ve$ and $2\sin(-26.6°) = -ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ±5

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\left\{(2x+1)+8x\right\}$	M1	1.1b
	$$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$$	A1	1.1b
		(4 marks)	

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Question	Scheme	Marks	AOs
4 (a)	$gf(x) = 3\ln e^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$gf(x) = fg(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
		(5 marks)	

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$

or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05 \times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\mathrm{g}$	A1	1.1b
		(2)	
(b)	States or uses $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$	M1	2.1
	$\dfrac{\mathrm{d}m}{\mathrm{d}t} = -0.05 \times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
		(4 marks)	

Notes:

(a)

M1: Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05 \times 0.5}$

A1: $m = 24.4\mathrm{g}$ An answer of $m = 24.4\mathrm{g}$ with no working would score both marks

(b)

M1: Applies the rule $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kx}\right) = k\,\mathrm{e}^{kx}$ in this context by stating or using $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$

A1: $\dfrac{\mathrm{d}m}{\mathrm{d}t} = -0.05 \times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$

Question	Scheme	Marks	AOs
6(i)	$x^2 - 6x + 10 = (x-3)^2 + 1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2 + 1 \geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a < 0$ then $ax > b \Rightarrow x < \dfrac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a > 0$ then $ax > b \Rightarrow x > \dfrac{b}{a}$ if $a < 0$ then $ax > b \Rightarrow x < \dfrac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $= (n+1)^2 - n^2 = 2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1 = (\text{even} +1) = \text{odd}$	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y = x^2 - 6x + 10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if

$a < 0$ then $ax > b \Rightarrow x < \dfrac{b}{a}$ or simply $-3x > 6 \Rightarrow x < -2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.

For example by attempting $(n+1)^2 - n^2 = 2n+1$ or $m^2 - n^2 = (m-n)(m+n)$ with $m = n+1$

A1: States always true with reason and proof

Accept a proof written in words. For example

If integers are consecutive, one is odd and one is even

When squared $\text{odd} \times \text{odd} = \text{odd}$ and $\text{even} \times \text{even} = \text{even}$

The difference between odd and even is always odd, hence always true

Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$$\sqrt{(4-x)} = 2\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}}$$	M1	2.1
	$$\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\left(-\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2 + ...$$	M1	1.1b
	$$\sqrt{(4-x)} = 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2 + ..\right)$$	A1	1.1b
	$$\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + ... \text{ and } k = -\frac{1}{64}$$	A1	1.1b
		(4)	
(b)	The expansion is valid for $|x| < 4$, so $x = 1$ can be used	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1 \pm ...)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n = \frac{1}{2}$

Eg. $(1 + ax)^{\frac{1}{2}} = 1 + \frac{1}{2}(ax) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2 + ...$

A1: Correct expression inside the bracket $1 - \frac{1}{8}x - \frac{1}{128}x^2 +$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + ...$ and $k = -\frac{1}{64}$

(b)

B1: The expansion is valid for $|x| < 4$, so $x = 1$ can be used

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\dfrac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\dfrac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ $\quad$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD $\quad$ Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\dfrac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\dfrac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
		(7 marks)	

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\dfrac{2}{5}$

B1: States that y coordinate of $A = 2$

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\dfrac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\dfrac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\dfrac{4}{5}\right)^2 + 2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5 + \dfrac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \dfrac{1}{2}BD \times AO = 2 \times \dfrac{1}{2}\,'5.8'\times'2'$

A1: Area $ABCD = 11.6$ or other exact equivalent such as $\dfrac{58}{5}$

Question	Scheme	Marks	AOs
9	$\displaystyle\int \left(3x^{0.5} + A\right)dx = 2x^{1.5} + Ax(+c)$	M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow (2\times 8 + 4A) - (2\times 1 + A) = 2A^2$	M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts $b^2 - 4ac$	M1	1.1b
	$\Rightarrow A = -2, \dfrac{7}{2}$ **and** states that there are two roots	States $b^2 - 4ac = 121 > 0$ and hence there are two roots	A1	2.4

(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5} + Ax(+c)$ where k is a non-zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\displaystyle\int A\,dx = Ax$ and not $\dfrac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts $b^2 - 4ac$

A1: Either $A = -2, \dfrac{7}{2}$ and states that there are two roots

Or states $b^2 - 4ac = 121 > 0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_\infty = \dfrac{8}{7} \times S_6 \Rightarrow \dfrac{a}{1-r} = \dfrac{8}{7} \times \dfrac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \dfrac{8}{7} \times (1 - r^6)$	M1	2.1
	$\Rightarrow r^6 = \dfrac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\dfrac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b

(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \dfrac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm\dfrac{1}{\sqrt{2}}$ giving $k = 2$

Question	Scheme	Marks	AOs
11 (a)	$\mathrm{f}(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5 = \dfrac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $$\dfrac{3}{2}x = 31$$	M1	1.1b
	$x = \dfrac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k > 5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R},\ 5 < k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

Notes:

(a)

B1: $\mathrm{f}(x) \geqslant 5$ Also allow $\mathrm{f}(x) \in [5, \infty)$

(b)

M1:   Deduces that the solution to $\mathrm{f}(x) = \dfrac{1}{2}x+30$ can be found by solving

$$-2(3-x)+5 = \dfrac{1}{2}x+30$$

M1:   Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x = \dfrac{62}{3}$ only. Do not allow 20.6

(c)

M1:   Deduces that two distinct roots occurs when $y = k$ intersects $y = \mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k > 5$ or $k \leqslant 11$

A1:   Correct solution only $\{k : k \in \mathbb{R},\ 5 < k \leqslant 11\}$

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \dfrac{1}{4}, -\dfrac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \dfrac{1}{4}, -\dfrac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° = $ their $'-19.47°'$

A1ft: $\theta = 5.26°$ but allow a follow through on their $'-19.47°'$

Question	Scheme	Marks	AOs
13(a)	$R = \sqrt{109}$	B1	1.1b
	$\tan\alpha = \dfrac{3}{10}$	M1	1.1b
	$\alpha = 16.70°$ so $\sqrt{109}\cos(\theta + 16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H = 11 - 10\cos(80t)° + 3\sin(80t)°$ or $H = 11 - \sqrt{109}\cos(80t + 16.70)°$	B1	3.3
	(ii) $11 + \sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t + "16.70" = 540$	M1	3.4
	$t = \dfrac{540 - "16.70"}{80} = (6.54)$	M1	1.1b
	$t = 6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H = 11 - 10\cos(90t)° + 3\sin(90t)°$		3.3
		(1)	
		(9 marks)	

Notes:

(a)

B1: $R = \sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha = \pm\dfrac{3}{10}$

A1: $\alpha = 16.70°$

(b)(i)
B1: see scheme

(b)(ii)
B1ft: their $11 +$ their $\sqrt{109}$ Allow decimals here.

(c)
M1: Sets $80t + "16.70" = 540$. Follow through on their 16.70
M1: Solves their $80t + "16.70" = 540$ correctly to find t
A1: $t = 6$ mins 32 seconds

(d)
B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $\quad S = 2\pi r^2 + \dfrac{1000}{r} \quad *$	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{\mathrm{d}S}{\mathrm{d}r}$	M1	3.4
	$\dfrac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 0$ and proceeds to $r^3 = k, \ k$ is a constant	M1	2.1
	Radius $= 4.30\,\mathrm{cm}$	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60\,\mathrm{cm}$	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3\,\mathrm{cm}$ and $h = 8.6\,\mathrm{cm}$ the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
		9 marks	

Notes:

(a)

B1: Uses the correct volume formula with $V = 500$. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{\mathrm{d}S}{\mathrm{d}r} = Ar \pm Br^{-2}$

A1: $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 0$ and proceeds to $r^3 = k, \ k$ is a constant

A1: $R =$ awrt 4.30cm

A1: $H =$ awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question	Scheme	Marks	AOs
15	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - \left(6x - 9\right)\mathrm{d}x$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \dfrac{15}{2}x^2 + 20x\,(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \dfrac{15}{2}x^2 + 20x\right]_0^4 = 2 \times 4^{\frac{5}{2}} - \dfrac{15}{2} \times 4^2 + 20 \times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
		(10 marks)	

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\dfrac{dy}{dx} = \dfrac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\dfrac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) - \left(6x - 9 \right) dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \dfrac{15}{2}x^2 + 20x \left(+c \right) \right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for
- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \displaystyle\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx \right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \dfrac{9}{2}x^2 + 11x \right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R =$ their $36 - \dfrac{1}{2} \times 15 \times \left(4 - \text{their } \dfrac{3}{2} \right) + \dfrac{1}{2} \times \text{their } 9 \times \text{their } \dfrac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP\Rightarrow A\ or\ B$	M1	1.1b
	$\dfrac{1}{P(11-2P)}=\dfrac{1/11}{P}+\dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int\dfrac{22}{P(11-2P)}\,dP=\int 1\,dt$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int\dfrac{2}{P}+\dfrac{4}{(11-2P)}\,dP=t+c$	M1	1.1b
	$2\ln P-2\ln(11-2P)=t+c$	A1	1.1b
	Substitutes $t=0,P=1\Rightarrow t=0,P=1\Rightarrow c=(-2\ln 9)$	M1	3.1a
	Substitutes $P=2\Rightarrow t=2\ln 2+2\ln 9-2\ln 7$	M1	3.1a
	Time $=1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $\quad 2\ln P-2\ln(11-2P)=t-2\ln 9$ $\Rightarrow\ln\left(\dfrac{9P}{11-2P}\right)=\dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow\left(\dfrac{9P}{11-2P}\right)=e^{\frac{1}{2}t}$ $\Rightarrow 9P=(11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P=f\left(e^{\frac{1}{2}t}\right)\ or\ \Rightarrow P=f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P=\dfrac{11}{2+9e^{-\frac{1}{2}t}}\Rightarrow A=11,B=2,C=9$	A1	1.1b
		(3)	
		(12 marks)	

Question 16 continued

Notes:

(a)

B1: Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P = \dfrac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A$ or B

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)} = \dfrac{1}{11P} + \dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\displaystyle\int \dfrac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$ or equivalent

M1: Uses part (a) and $\displaystyle\int \dfrac{A}{P} + \dfrac{B}{(11-2P)}\mathrm{d}P = A\ln P \pm C\ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg
$2\ln P - 2\ln(11-2P) = t + c$

M1: Substitutes $t = 0$ and $P = 1$ to find c

M1: Substitutes $P = 2$ to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P - 2\ln(11-2P) = t + c$ to $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d$

for their numerical 'c'

M1: Uses a correct method to get P in terms of $e^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d \Rightarrow \dfrac{P}{11-2P} = e^{\frac{1}{2}t+d}$ followed by cross

multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $e^{-\frac{1}{2}t}$ For example

$\dfrac{P}{11-2P} = e^{\frac{1}{2}t+d} \Rightarrow \dfrac{11-2P}{P} = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} - 2 = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} = 2 + e^{-\left(\frac{1}{2}t+d\right)}$ followed by
division

A1: Achieves the correct answer in the form required. $P = \dfrac{11}{2 + 9e^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$ oe