IYGB – MP2 PAPER Q – QUESTION 1

ASSERTION: IF $|2x+1| \leq 5$, THEN $|x| \leq 2$

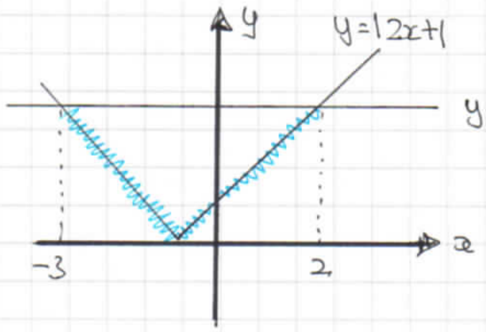
- EITHER WE USE A BIT OF COMMON SENSE AND PICK A SENSIBLE NUMBER SUCH AS $x = -\frac{5}{2}$

$$|2\times(-\tfrac{5}{2})+1| = |-5+1| = |-4| = 4 \leq 5$$

BUT

$$|-\tfrac{5}{2}| = \tfrac{5}{2} \geqslant 2$$ WHICH DISPROVES IT

- OR WE FIND THE SOLUTION INTERVAL FOR THE MODULUS INEQUALITY



$$\left.\begin{array}{l} 2x+1 = 5 \\ 2x+1 = -5 \end{array}\right\} \Rightarrow \begin{array}{l} x = 2 \\ x = -3 \end{array}$$

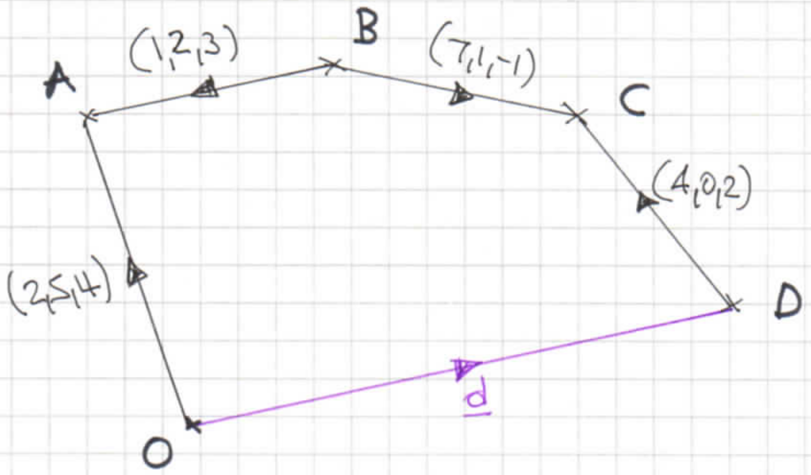
HENCE THE SOLUTION INTERVAL IS $-3 \leq x \leq 2$,

I.E $|x| \leq 2 \cup -3 \leq x < -2$

SO THE ASSERTION IS FALSE

IYGB – MP2 PAPER Q – QUESTION 2

STARTING WITH A VECTOR DIAGRAM



$A(2,5,4)$
$\overrightarrow{BA} = (1,2,3)$
$\overrightarrow{BC} = (7,1,-1)$
$\overrightarrow{DC} = (4,0,2)$

FORMING A VECTOR EQUATION TO FIND $\overrightarrow{OD}$

$$\Longrightarrow \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$$

$$\Longrightarrow \overrightarrow{OD} = (2,5,4) - (1,2,3) + (7,1,-1) - (4,0,2)$$

$$\Longrightarrow \overrightarrow{OD} = (4,4,-2)$$

I.E $\underline{D(4,4,-2)}$

FINALLY THE DISTANCE OF D FROM THE ORIGIN

$$\Longrightarrow |\overrightarrow{OD}| = |4,4,-2|$$

$$\Longrightarrow |\overrightarrow{OD}| = \sqrt{4^2+4^2+(-2)^2}$$

$$\Longrightarrow |\overrightarrow{OD}| = \sqrt{16+16+4}$$

$$\Longrightarrow \underline{|\overrightarrow{OD}| = 6}$$

IYGB - MP2 PAPER Q - QUESTION 3

COLLECTING ALL THE RELEVANT INFORMATION

$$\frac{dA}{dt} = -kA$$

RATE

DECREASE

PROPORTIONAL

AREA STILL COVERED BY WEED

$t=0 \quad A=75$

$t=2 \quad A=33.7$

SOLVE BY SEPARATION OF VARIABLES

$$\Rightarrow \quad dA = -kA\,dt$$

$$\Rightarrow \quad \frac{1}{A}\,dA = -k\,dt$$

$$\Rightarrow \quad \int \frac{1}{A}\,dA = \int -k\,dt$$

$$\Rightarrow \quad \ln A = -kt + C \qquad (A>0)$$

$$\Rightarrow \quad A = e^{-kt+C}$$

$$\Rightarrow \quad A = e^{-kt} \times e^{C}$$

$$\Rightarrow \quad A = Ce^{-kt} \qquad (e^{C} = C)$$

APPLY CONDITIONS

$$t=0,\ A=75 \quad \Rightarrow \quad 75 = Ce^{0}$$

$$\Rightarrow \quad 75 = C$$

$$\Rightarrow \quad A = 75e^{-kt}$$

IYGB – MP2 PAPER Q – QUESTION 3

WHEN $t=2$ $A=33.7$ $\Longrightarrow$ $33.7 = 75e^{-k \times 2}$

$$\Longrightarrow e^{-2k} = \frac{337}{750}$$

$$\Longrightarrow e^{2k} = \frac{750}{337}$$

$$\Longrightarrow 2k = \ln\left(\frac{750}{337}\right)$$

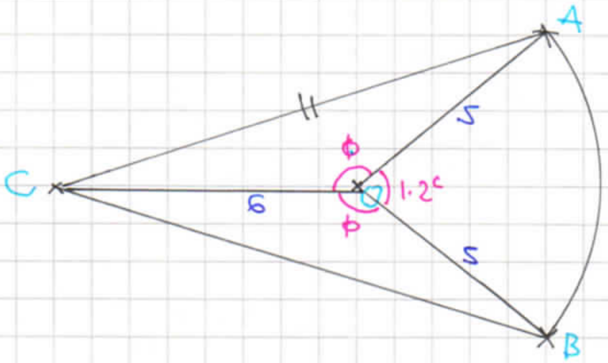
$$\Longrightarrow k = \frac{1}{2}\ln\left(\frac{750}{337}\right)$$

$$\Longrightarrow k = 0.3999995\ldots$$

$$\therefore A = 75e^{-0.4t}$$

IYGB - MP2 PAPER Q - QUESTION 4

a) LOOKING AT THE DIAGRAM BELOW



AREA OF SECTOR $\hat{AOB}$ $= "\frac{1}{2}r^2\theta^c" = \frac{1}{2} \times 5^2 \times 1.2 = 15 \text{ cm}^2$

b) THE TRIANGLES COA & COB ARE IDENTICAL (SSS CASE)

$\therefore \hat{COA} = \hat{COB} \implies 2\phi + 1.2^c = 2\pi$

$\implies \phi + 0.6^c = \pi$

$\implies \phi = \pi - 0.6^c$

$\implies \phi = 2.54159...^c$

$\therefore \phi \simeq 2.54^c$

c) AREA OF $\hat{COA}$ IS GIVEN BY $\frac{1}{2}|OC||OA|\sin\phi$

$\implies \frac{1}{2} \times 6 \times 5 \times \sin(2.54...^c) = 8.469.... \text{ cm}^2$

THE REQUIRED AREA IS

$2 \times 8.469 + 15 = 31.9 \text{ cm}^2$

IYGB - MP2 PAPER Q - QUESTION 5

DETERMINE AN EXPRESSION FOR THE INTEGRAL IN TERMS OF k

$$\int_0^{\frac{\pi}{3}} (k\cos^2 x - \sec^2 x)\sin x \, dx = \int_0^{\frac{\pi}{3}} k\cos^2 x\sin x - \sec^2 x\sin x \, dx$$

$$= \int_0^{\frac{\pi}{3}} k\cos^2 x\sin x - \sec x \times \frac{1}{\cos x} \times \sin x \, dx$$

$$= \int_0^{\frac{\pi}{3}} k\cos^2 x\sin x - \sec x\tan x \, dx$$

BY RECOGNITION WE OBTAIN

$$= \left[-\frac{k}{3}\cos^3 x - \sec x\right]_0^{\frac{\pi}{3}} = \left[\frac{k}{3}\cos^3 x + \sec x\right]_{\frac{\pi}{3}}^{0}$$

$$= \left(\frac{k}{3} + 1\right) - \left(\frac{k}{24} + 2\right) = \frac{k}{3} - \frac{k}{24} - 1$$

$$= \frac{1}{24}(8k - k - 24) = \frac{1}{24}(7k - 24)$$

FINALLY WE HAVE

$$\frac{1}{24}(7k - 24) = \frac{3}{4}$$

$$7k - 24 = 18$$

$$7k = 42$$

$$k = 6$$

IYGB – MP2 PAPER Q – QUESTION 6

a) $f(x) = \sqrt{1-x} = (1-x)^{\frac{1}{2}}$

$$= 1 + \frac{\frac{1}{2}}{1}(-x)^1 + \frac{\frac{1}{2}(-\frac{1}{2})}{1\times2}(-x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\times2\times3}(-x)^3 + \dots$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + O(x^4)$$

b) BY DIRECT PROOF

$$7\sqrt{1-\frac{1}{49}} = 7\sqrt{\frac{48}{49}} = \cancel{7}\frac{\sqrt{48}}{\cancel{\sqrt{49}}} = \sqrt{48}$$

$$= \sqrt{16\times3} = 4\sqrt{3}$$

c) FROM PART (a)

$$\Longrightarrow \sqrt{1-x} \approx 1 - \frac{1}{2}x$$

$$\Longrightarrow 7\sqrt{1-x} \approx 7\left(1-\frac{1}{2}x\right)$$

$$\Longrightarrow 7\sqrt{1-x} \approx 7 - \frac{7}{2}x$$

LET $x = \frac{1}{49}$ (AS IN PART b)

$$\Longrightarrow 7\sqrt{1-\frac{1}{49}} \approx 7 - \frac{7}{2}\left(\frac{1}{49}\right)$$

$$\Longrightarrow 4\sqrt{3} \approx 7 - \frac{7}{98}$$

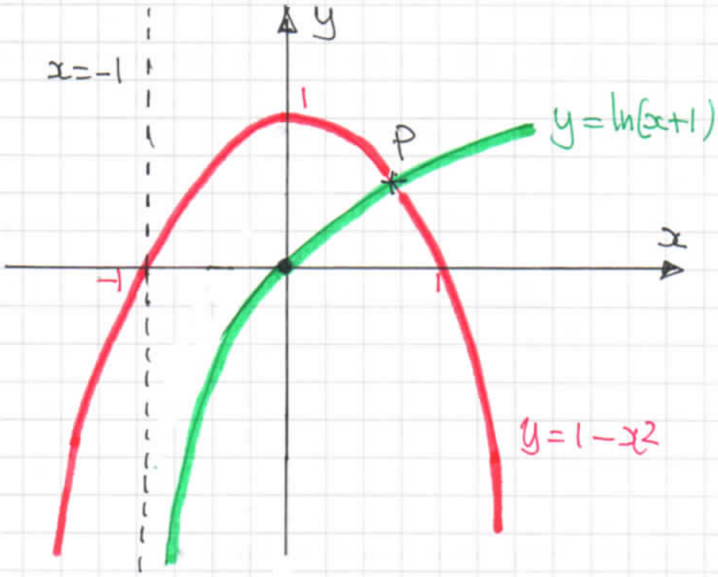
$$\Longrightarrow 4\sqrt{3} \approx \frac{97}{14}$$

$$\Longrightarrow \sqrt{3} \approx \frac{97}{56}$$

AS REQUIRED

IYGB - MP2 PAPER Q - QUESTION 7

a) USING A QUICK SKETCH OF THE TWO GRAPHS



ONE INTERSECTION AS SEEN ABOVE - AS THE x INTERCEPTS OF $y = 1-x^2$ & $\ln(x+1)$ ARE 1 & 0, THE x CO-ORDINATE OF P MUST LIE BETWEEN THEM

b) FORM AN EQUATION & WRITE IT AS A FUNCTION

$$\Rightarrow \ln(1+x) = 1 - x^2$$

$$\Rightarrow x^2 - 1 + \ln(1+x) = 0$$

$$\Rightarrow f(x) = x^2 - 1 + \ln(1+x)$$

$$\Rightarrow f'(x) = 2x + \frac{1}{x+1}$$

$$\therefore \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x \simeq 0.7 - \frac{(0.7)^2 - 1 + \ln(1+0.7)}{2\times0.7 + \frac{1}{0.7+1}} \simeq 0.690$$

IYGB – MP2 PAPER Q – QUESTION 7

c) LOOKING AT THE TWO GRAPHS, THEY HAVE BOTH UNDERGONE IDENTICAL TRANSFORMATIONS

- TRANSLATION, 1 UNIT, "LEFT"
- HORIZONTAL STRETCH, SCALE FACTOR $\frac{1}{2}$
- VERTICAL STRETCH, SCALE FACTOR 2

IF $x \simeq 0.690 \implies y \simeq 0.524$

$$\therefore (0.690, 0.524)$$
$$\downarrow$$
$$(-0.310, 0.524)$$
$$\downarrow$$
$$(-0.155, 0.524)$$
$$\downarrow$$
$$(-0.155, 1.048)$$

$\therefore (-0.16, 1.05)$ //

IYGB – MP2 PAPER Q – QUESTION 8

START BY APPLYING THE COMPOUND ANGLE IDENTITY FOR $\tan(A-B)$

$$\Rightarrow \tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$$

$$= \frac{4\tan\theta - 4\tan\phi}{4+4\tan\theta\tan\phi}$$

BUT $4\tan\phi = 3\tan\theta$

$$= \frac{4\tan\theta-(3\tan\theta)}{4+\tan\theta(3\tan\theta)}$$

$$= \frac{4\tan\theta - 3\tan\theta}{4+3\tan^2\theta}$$

$$= \frac{\tan\theta}{4+3\tan^2\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{4+\frac{3\sin^2\theta}{\cos^2\theta}}$$

MULTIPLY "TOP & BOTTOM" OF THE DOUBLE FRACTION BY $\cos^2\theta$

$$= \frac{\sin\theta\cos\theta}{4\cos^2\theta+3\sin^2\theta}$$

$$= \frac{2\sin\theta\cos\theta}{8\cos^2\theta+6\sin^2\theta}$$

$$= \frac{\sin2\theta}{8\left(\frac{1}{2}+\frac{1}{2}\cos2\theta\right)+6\left(\frac{1}{2}-\frac{1}{2}\cos2\theta\right)}$$

$\cos2\theta = 2\cos^2\theta-1$
$\cos2\theta = 1-2\sin^2\theta$

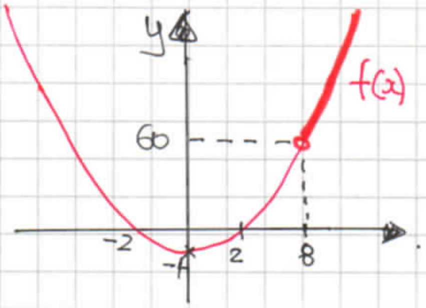
$$= \frac{\sin2\theta}{4+4\cos2\theta+3-3\cos2\theta}$$

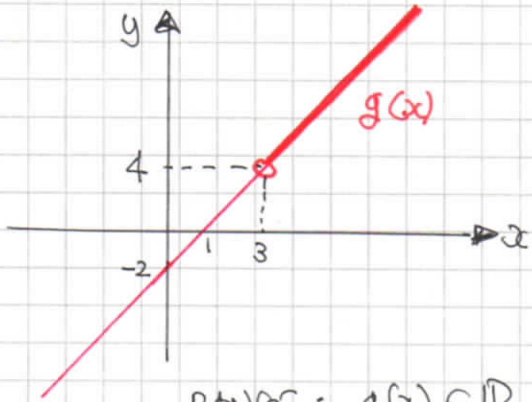
$$= \frac{\sin2\theta}{7+\cos2\theta}$$

// AS REQUIRED

IYGB - MP2 PAPER Q - QUESTION 9

a) BY SKETCHING OR VISUALIZING THE GRAPHS





RANGE: $f(x) \in \mathbb{R}$
$f(x) > 60$

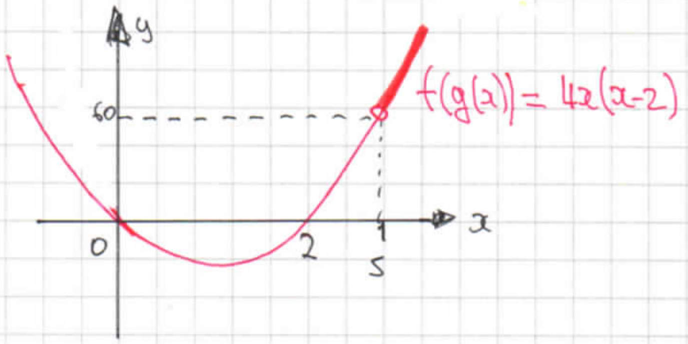
RANGE: $g(x) \in \mathbb{R}$
$g(x) > 4$

b) $f(g(x)) = f(2x-2) = (2x-2)^2 - 4 = 4x^2 - 8x$

c) DRAWING A DIAGRAM

$x > 3$ $2x-2$ $x > 8$ $4x^2-8x$

$2x - 2 > 8$
$x > 5$



$\therefore$ DOMAIN $x > 5$

RANGE $f(g(x)) > 60$

IYGB – MP2 PAPER Q – QUESTION 10

a) START BY RELATING DERIVATIVES

$$\Rightarrow \frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \times 20 \leftarrow \text{GIVEN}$$

$$\Rightarrow \boxed{\frac{dr}{dt} = \frac{5}{\pi r^2}}$$

VOLUME OF A SPHERE

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dV} = \frac{1}{4\pi r^2}$$

$$\Rightarrow \left.\frac{dr}{dt}\right|_{r=5} = \frac{5}{\pi \times 5^2} = \frac{1}{5\pi} \approx 0.0637 \text{ cm s}^{-1}$$

b) WE NEED TO FIND THE RADIUS OF THE BUBBLE WHEN ITS VOLUME REACHES 300 cm^3

$$\Rightarrow V = \frac{4}{3}\pi r^3$$

$$\Rightarrow 300 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{225}{\pi} = r^3$$

$$\Rightarrow r = \sqrt[3]{\frac{225}{\pi}} \approx 4.1528\ldots$$

$$\Rightarrow \left.\frac{dr}{dt}\right|_{r=4.1528\ldots} = \frac{5}{\pi \times (4.1528\ldots)^2} \approx 0.0923 \text{ cm s}^{-1}$$

c) WE NEED TO CHANGE THE "TIME" INTO "VOLUME" FIRST

"... CONSTANT RATE OF 20 cm^3 PER SECOND ..."

$\therefore$ IN 10 SECONDS THE VOLUME WILL BE $20 \times 10 = 200 \text{ cm}^3$

IYGB - MP2 PAPER Q - QUESTION 10

PROCEED AS IN PART (b)

$$\Rightarrow V = \frac{4}{3}\pi r^3$$

$$\Rightarrow 200 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{150}{\pi} = r^3$$

$$\Rightarrow r = \sqrt[3]{\frac{150}{\pi}} \simeq 3.6278\ldots$$

$$\Rightarrow \left.\frac{dr}{dt}\right|_{r=3.6278\ldots} = \frac{5}{\pi(3.6278\ldots)^2} \approx 0.121 \text{ cm s}^{-1}$$

IYGB - MP2 PAPER Q - QUESTION 11

a) USING THE QUOTIENT RULE

$$y = \frac{e^x}{\sin x} \Longrightarrow \frac{dy}{dx} = \frac{(\sin x)e^x - e^x\cos x}{(\sin x)^2}$$

$$\Longrightarrow \frac{dy}{dx} = \frac{e^x(\sin x - \cos x)}{\sin^2 x}$$

$$\Longrightarrow \frac{dy}{dx} = \frac{e^x}{\sin x}\left(\frac{\sin x - \cos x}{\sin x}\right)$$

$$\Longrightarrow \frac{dy}{dx} = y\left(\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$\Longrightarrow \frac{dy}{dx} = y(1 - \cot x)$$

AS REQUIRED

b) DIFFERENTIATE THE RESULT OF PART (a) WITH RESPECT TO x

$$\Longrightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[y(1-\cot x)\right]$$ ← PRODUCT RULE

$$\Longrightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}(1-\cot x) + y\frac{d}{dx}(1-\cot x)$$

$$\Longrightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}(1-\cot x) + y\,\mathrm{cosec}^2 x$$

AS REQUIRED

c) SOLVE $\frac{dy}{dx} = 0$ TO GET

$\Longrightarrow y(1-\cot x) = 0$

$\Longrightarrow 1 - \cot x = 0$ SINCE $y = \frac{e^x}{\sin x} \neq 0$

$\Longrightarrow \cot x = 1$

$\Longrightarrow \tan x = 1$

$\Longrightarrow x = \frac{\pi}{4}$ (ONLY SOLUTION $0 < x < \pi$)

$$\& \quad y = \frac{e^{\pi/4}}{\sin\pi/4} = \frac{e^{\pi/4}}{1/\sqrt{2}} = \sqrt{2}e^{\frac{\pi}{4}}$$

IYGB - MP2 PAPER Q - QUESTION 11

USING THE SECOND DERIVATIVE TO INVESTIGATE THE NATURE

$$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=\frac{\pi}{4} \\ \frac{dy}{dx}=0 \\ y=\sqrt{2}e^{\frac{\pi}{4}}}} = 0 + \sqrt{2}e^{\frac{\pi}{4}} \times \operatorname{cosec}^2\frac{\pi}{4} > 0$$

$\therefore$ (LOCAL) MINIMUM AT $\left(\frac{\pi}{4}, \sqrt{2}e^{\frac{\pi}{4}}\right)$

IYGB - MP2 PAPER Q - QUESTION 12

a) START BY OBTAINING THE GRADIENT FUNCTION IN PARAMETRIC

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-3\sin t} = -\frac{4\cos t}{3\sin t}$$

$$\left.\frac{dy}{dx}\right|_{t=\theta} = -\frac{4\cos\theta}{3\sin\theta} \quad \text{AT POINT } (3\cos\theta, 4\sin\theta)$$

EQUATION OF TANGENT IS GIVEN BY

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 4\sin\theta = -\frac{4\cos\theta}{3\sin\theta}(x - 3\cos\theta)$$

$$\Rightarrow 3y\sin\theta - 12\sin^2\theta = -4x\cos\theta + 12\cos^2\theta$$

$$\Rightarrow 3y\sin\theta + 4x\cos\theta = 12\cos^2\theta + 12\sin^2\theta$$

$$\Rightarrow 3y\sin\theta + 4x\cos\theta = 12(\cancel{\cos^2\theta + \sin^2\theta})$$

$$\Rightarrow 3y\sin\theta + 4x\cos\theta = 12$$

AS REQUIRED

b) SETTING $x=0$ IN THE EQUATION OF THE TANGENT YIELDS $P(0,8)$

$$\Rightarrow 3 \times 8\sin\theta + 0 = 12$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \begin{cases} \pi/6 \\ 5\pi/6 \end{cases}$$

EQUATION OF TANGENT BECOMES

$$3y\sin\frac{\pi}{6} + 4x\cos\frac{\pi}{6} = 12 \quad \text{OR} \quad 3y\sin\frac{5\pi}{6} + 4x\cos\frac{5\pi}{6} = 12$$

$$\frac{3}{2}y + 2\sqrt{3}x = 12 \quad \quad \frac{3}{2}y - 2\sqrt{3}x = 12$$

SOLVING EACH EQUATION FOR $y=0$ TO OBTAIN Q

$$2\sqrt{3}x = 12 \quad \quad -2\sqrt{3}x = 12$$

$$x = \frac{6}{\sqrt{3}} \quad \text{OR} \quad x = -\frac{6}{\sqrt{3}}$$

IYGB - MP2 PAPER Q - QUESTION 12

$\therefore \; x = \pm 2\sqrt{3}$ $\quad$ # $\quad Q(\pm 2\sqrt{3}, 0)$

FINALLY AREA CAN BE FOUND

$$\text{AREA} = \frac{1}{2} \times 8 \times |\pm 2\sqrt{3}|$$

$$\text{AREA} = 4 \times 2\sqrt{3}$$

$$\text{AREA} = 8\sqrt{3}$$

IYGB - MP2 PAPER Q - QUESTION 13

MANIPULATE THE SUMS AS FOLLOWS

$$\sum_{r=1}^{20}(f(r)-10) = \left[\sum_{r=1}^{20} f(r)\right] - \sum_{r=1}^{20} 10$$

$$200 = \left[\sum_{r=1}^{20} f(r)\right] - 10 \times 20$$

$$\sum_{r=1}^{20} f(r) = 400$$

NEXT WE HAVE

$$\sum_{r=1}^{20}(f(r)-10)^2 = \sum_{r=1}^{20}\left[[f(r)]^2 - 20f(r) + 100\right]$$

$$2800 = \sum_{r=1}^{20}[f(r)]^2 - 20\sum_{r=1}^{20}[f(r)] + 100\sum_{r=1}^{20} 1$$

$$2800 = \sum_{r=1}^{20}(f(r))^2 - 20 \times 400 + 100 \times 20$$

$$2800 = \sum_{r=1}^{20}(f(r))^2 - 6000$$

$$\sum_{r=1}^{20}[f(r)]^2 = 8800$$