

**FP2 Further complex numbers Questions**

3. (a) Express the complex number  $-2 + (2\sqrt{3})i$  in the form  $r(\cos \theta + i \sin \theta)$ ,  $-\pi < \theta \leq \pi$ .  
(3)

- (b) Solve the equation

$$z^4 = -2 + (2\sqrt{3})i$$

giving the roots in the form  $r(\cos \theta + i \sin \theta)$ ,  $-\pi < \theta \leq \pi$ .  
(5)

8. The point  $P$  represents a complex number  $z$  on an Argand diagram such that

$$|z - 6i| = 2|z - 3|$$

- (a) Show that, as  $z$  varies, the locus of  $P$  is a circle, stating the radius and the coordinates of the centre of this circle.  
(6)

The point  $Q$  represents a complex number  $z$  on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}$$

- (b) Sketch, on the same Argand diagram, the locus of  $P$  and the locus of  $Q$  as  $z$  varies.  
(4)

- (c) Find the complex number for which both  $|z - 6i| = 2|z - 3|$  and  $\arg(z - 6) = -\frac{3\pi}{4}$   
(4)

5. The point  $P$  represents the complex number  $z$  on an Argand diagram, where

$$|z - i| = 2$$

The locus of  $P$  as  $z$  varies is the curve  $C$ .

- (a) Find a cartesian equation of  $C$ .

(2)

- (b) Sketch the curve  $C$ .

(2)

A transformation  $T$  from the  $z$ -plane to the  $w$ -plane is given by

$$w = \frac{z + i}{3 + iz}, \quad z \neq 3i$$

The point  $Q$  is mapped by  $T$  onto the point  $R$ . Given that  $R$  lies on the real axis,

- (c) show that  $Q$  lies on  $C$ .

(5)

6. A complex number  $z$  is represented by the point  $P$  in the Argand diagram.

- (a) Given that  $|z - 6| = |z|$ , sketch the locus of  $P$ .

(2)

- (b) Find the complex numbers  $z$  which satisfy both  $|z - 6| = |z|$  and  $|z - 3 - 4i| = 5$ .

(3)

The transformation  $T$  from the  $z$ -plane to the  $w$ -plane is given by  $w = \frac{30}{z}$ .

- (c) Show that  $T$  maps  $|z - 6| = |z|$  onto a circle in the  $w$ -plane and give the cartesian equation of this circle.

(5)

4. The point  $P$  represents a complex number  $z$  on an Argand diagram such that

$$|z - 3| = 2|z|.$$

- (a) Show that, as  $z$  varies, the locus of  $P$  is a circle, and give the coordinates of the centre and the radius of the circle.

(5)

The point  $Q$  represents a complex number  $z$  on an Argand diagram such that

$$|z + 3| = |z - i\sqrt{3}|.$$

- (b) Sketch, on the same Argand diagram, the locus of  $P$  and the locus of  $Q$  as  $z$  varies.

(5)

- (c) On your diagram shade the region which satisfies

$$|z - 3| \geq 2|z| \quad \text{and} \quad |z + 3| \geq |z - i\sqrt{3}|.$$

(2)

2.

$$z = 5\sqrt{3} - 5i$$

Find

- (a)  $|z|$ ,

(1)

- (b)  $\arg(z)$ , in terms of  $\pi$ .

(2)

$$w = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Find

- (c)  $\left|\frac{w}{z}\right|$ ,

(1)

- (d)  $\arg\left(\frac{w}{z}\right)$ , in terms of  $\pi$ .

(2)

