Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics

AS Further Statistics 1 Paper 8FM0_23

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu	Scheme	Marks	AO
1(a)	H_0 : There is no association between age and activity H_1 : There is an association between age and activity	B1 (1)	1.2
(b)	$\frac{26\times 36}{150} = 6.24$	B1 (1)	1.1b
(c)	Since expected value in a < 20 and snooker = 3.6 < 5 we amalgamate two rows Table is now 4×3 so degrees of freedom is $(4-1)\times(3-1)=6$ (*****)	B1 B1* (2)	2.4 1.1b
(d)	Critical value $\chi_6^2(5\%)=12.592$ [Significant result]: so there is evidence to support manager's belief	B1 B1ft (2)	1.1b 2.2b
		(6 marks)	

Notes

(a) B1 for both hypotheses in terms of "association" or "independence". Must mention age and activity (or sport). [Use of "relationship" or "link" here is B0 but allow for last B1ft]

(b) B1 for 6.24

(c) 1st B1 for a reason to get a 4 x 3 table based on amalgamation of rows
Must mention a < 20 and snooker and see 3.6 and be combining <u>rows</u> (not columns)
2nd B1* for 6 degrees of freedom clearly coming from a 4 x 3 table formed from amalgamation of <u>rows</u>. [8 – 2 = 6 is B0]

(d) 1st B1 for correct critical value (allow 12.6 or 12.59 or awrt 12.592)
NB p-value = 0.0032839… so allow awrt 0.00328
2nd B1ft for a correct comparison and conclusion (ft their cv) [<u>Independent of hypotheses</u>]
e.g. there is an "association" or "relationship" or "link" between age and activity is OK
BUT there is a "correlation" between age and activity is B0
Do not accept contradictory contextual statements e.g. "manager's belief supported , there is no association between age and activity"

Qu	Scheme	Marks	AO
2.	H_0: Spinner is working as designed (o.e.) H_1: Spinner is not working as designed (o.e.)	B1	1.2
	E_i | 24 | 8 | 8 | 32 | 8 O_i | 15 | 4 | 12 | 41 | 8 $\frac{(O_i-E_i)^2}{E_i}$ | $\frac{81}{24}$ | $\frac{16}{8}$ | $\frac{16}{8}$ | $\frac{81}{32}$ | 0 $\frac{O_i^2}{E_i}$ | $\frac{225}{24}$ | $\frac{16}{8}$ | $\frac{144}{8}$ | $\frac{1681}{32}$ | $\frac{64}{8}$	M1 A1 M1	3.4 1.1b 1.1b
	$\sum \frac{(O_i-E_i)^2}{E_i} = 3.375+2+2+2.53125+0 = 9.90625$ or $\sum \frac{O_i^2}{E_i} - N = 9.375 + 2 + 18 + 52.53125 + 8 - 80 = 9.90625$	A1	1.1b
	$\nu = 5-1 = 4$ so $\chi_4^2(10\%)$ cv = 7.779 or better	B1	3.4
	Result is significant so there is evidence that the spinner is not operating as designed	A1cso **(7 marks)**	3.5a

Notes

1st B1 for both hypotheses given in suitable context
1st M1 for using the model to find at least 2 correct expected frequencies
1st A1 for all correct E_i
2nd M1 for attempt to find test statistic (at least two correct expressions, fractions or decimals)
2nd A1 for a correct test statistic (awrt 9.91) [accept $\frac{317}{32}$]
2nd B1 for correct critical value (allow 7.78)
NB *p*- value = 0.042036…so allow awrt 0.042
3rd A1cso dep on all previous marks for a correct conclusion in context
(can be in terms of model or spinner's design) Must mention spinner **and** scores or design
Accept "spinner is not accurate"

Qu	Scheme	Marks	AO
3(a)	[X = number of errors in 100-word piece] $X \sim$ Po(1.7)	M1	3.3
	P($X < 2$) = P($X \leqslant 1$) = 0.49324... awrt **0.493**	A1	1.1b
		(2)	
(b)	[R = number of errors in the article] $R \sim$ Po(4.25)	M1	3.3
	P($R = 5$) = 0.16482… awrt **0.165**	A1	1.1b
		(2)	
(c)	Scheme **A**: Let $A \sim$ B(40, $e^{-1.7}$) or B(40, 0.18268…)	M1	3.3
	P($A > 10$) = 1 – P($A \leqslant 10$)	M1	1.1b
	= 0.0995591… awrt **0.0996**	A1	1.1b
	Scheme **B** : Let $B \sim$ Po(40×1.7) or Po(68)	M1	3.3
	P($B < 56$) = P($B \leqslant 55$) = 0.061133…		
	So choose scheme **A** (since the probability of a bonus is greater)	A1	2.4
		(5)	
(d)	$H_0 : \lambda = 1.7$ (or $\mu = 8.5$) $H_1 : \lambda < 1.7$ (or $\mu < 8.5$)	B1	2.5
	[E = no. of errors in the piece of work] $E \sim$ Po(8.5)	M1	3.3
	P($E \leqslant 3$) = 0.0301 or P($E \leqslant 4$) = 0.0744	A1	1.1b
	So critical region is $\boldsymbol{E \leqslant 3}$	A1	2.2a
		(4)	
		(13 marks)	

Notes

(a)
M1 for selecting the correct Poisson distribution
A1 for awrt 0.493

(b)
M1 for selecting the correct Poisson distribution
A1 for awrt 0.165

(c)
1st M1 for choosing a correct model for scheme **A** i.e. B(40, P($X = 0$)), where $X \sim$Po(1.7)
Allow use of awrt 0.183 for P($X = 0$) … 0.183 gives answer awrt 0.101
Condone B(0.183, 40) (o.e.) if it leads to a prob rounding to range (0.09~0.1) otherwise M0
2nd M1 for 1 – P($A \leqslant 10$)
1st A1 for awrt 0.0996 [NB use of 0.183 will give awrt 0.101 and scores M1M1A0]
3rd M1 for selecting a correct Poisson model for scheme **B** i.e. Po(40×1.7) or better
2nd A1 for a correct conclusion based on comparing two probs: awrt 0.1 vs 0.061 or better
So can allow 0.1 > 0.061 leading to choosing **A** [Probably scores M1M1A0M1A1]

NB [Normal approx.(not on spec) leading to 0.06477… might score 3rd M1 if Po(68) seen but 2nd A0]

(d)
B1 for both hypotheses in terms of λ or μ (can be interchanged)
M1 for selecting Po(8.5) (sight of or use of e.g. may be implied by 1st A1)
1st A1 for some evidence of correct use of Po(8.5) i.e. either of these probs (2dp or better)
May be implied by a correct critical region
2nd A1 for a correct critical region. Allow $E < 4$ and allow any letter for E.
Two different regions (e.g. from 2 tail test) is 2nd A0

SC **Use of binomial throughout:** (with hypotheses $H_0 : p = 0.017$ and $H_1 : p < 0.017$ in (d))
Scores 0 in (a) 0 in (b) possibly just 2nd M1 in (c) But allow all 4 marks in (d): B1 hypotheses, M1 for $Y \sim$B(500, 0.017), 1st A1 for P($Y \leqslant 3$) = 0.02913…or P($Y \leqslant 4$) = 0.07266…2nd A1 $Y \leqslant 3$
Allow probs to be to 2dp or better so 0.03 and 0.07 as in main scheme.

Qu	Scheme	Marks	AO
4(a)	$\underline{q+\frac{7}{30}}$	B1	1.1b
		(1)	
(b)	$\mathrm{E}\left(X^2\right)=(-3)^2\times q \ + \ (-1)^2\times\frac{7}{30} \ + \ 1^2\times\frac{7}{30} \ + \ 2^2\times q \ + \ 4^2\times r$	M1	1.1b
	$= \underline{\frac{7}{15}+13q+16r}$ **(*)**	A1*cso	1.1b
		(2)	
(c)	$\mathrm{E}(X)=-3q \ + \ -\frac{7}{30} \ + \ \frac{7}{30} \ + \ 2q \ + \ 4r \quad \{=4r-q\}$	M1	3.1a
	$\mathrm{E}\left(X^2+6X\right)=\frac{7}{15}+7q+40r$	A1	1.1b
	$\mathrm{E}\left(X^3\right)=(-3)^3\times q \ + \ (-1)^3\times\frac{7}{30} \ + \ 1^3\times\frac{7}{30} \ + \ 2^3\times q \ + \ 4^3\times r$	M1	3.4
	$= 64r-19q$	A1	1.1b
	Sum of probabilities = 1 gives: $2q+r=\frac{16}{30}$ (o.e.)	M1	1.1b
	Solve: $24r-26q=\frac{7}{15}$ and $r+2q=\frac{8}{15}$ e.g. $37r=\frac{111}{15}$	dM1	1.1b
	So $\underline{r=\frac{1}{5}}$ and $\underline{q=\frac{1}{6}}$	A1	1.1b
		(7)	
(d)	$X^3>X^2+6X \ \Rightarrow X(X-3)(X+2)>0$	M1	2.1
	Use of sketch or table to see: $-2<X<0$ or $X>3$	A1	1.1b
	So $\mathrm{P}(X^3>X^2+6X)=\mathrm{P}(X=-1 \text{ or } 4)$	M1	2.2a
	$= \frac{7}{30}+"r"= \underline{\frac{13}{30}}$	A1ft	1.1b
		(4)	
ALT		**(14 marks)**	

X	-3	-1	1	2	4
X^3	–27	–1	1	8	64
X^2+6X	–9	–5	7	16	40

Notes

(b)

M1 for at least 3 correct terms of the expression for E(X^2)

A1*cso evidence of M1 scored with no incorrect working seen leading to correct answer **(*)**

Allow $-3^2\times q+-1^2\times\frac{7}{30}$ etc if followed by $9q$ + …but <u>not</u> if simply followed by given answer

(c)

1st M1 for realising the need to find E(X) – a correct attempt with at least 3 correct terms

1st A1 for the correct expression (needn't be simplified at this stage)

2nd M1 for a correct attempt at E(X^3) with at least 3 correct terms seen

Treat no $\frac{7}{30}$ terms as <u>one</u> correct term

2nd A1 for $64r-19q$ (must be simplified) <u>or for</u> $24r-26q=\frac{7}{15}$

3rd M1 for using sum of probabilities = 1 to form an equation in q and r (needn't be simplified)

Must be correct or clearly state that Σprobs = 1 being attempted with only one slip

4th dM1 for solving their 2 linear equations in q and r (dep on 3rd M1 and 1st <u>or</u> 2nd M1)

Must see correct method to reduce to a linear equation in one variable

3rd A1 for $r=\frac{1}{5}$ <u>and</u> $q=\frac{1}{6}$ or any exact equivalents (dep on 2 correct equations seen)

(d)

1st M1 for 1st stage towards solving the inequality (factorising the cubic)

1st A1 for solving the inequality

2nd M1 for identifying the values of X required i.e. – 1 and 4

2nd A1ft for $\frac{13}{20}$ or exact equivalent e.g. $0.4\dot{3}$ (Allow ft of "their r" + $\frac{7}{30}$)

ALT

Table 1st M1 for at least 4 correct values for X^3 <u>and</u> X^2+6X (must be labelled)

1st A1 for all 10 correct values. [NB Can score M1A0M1A1ft in (d)]

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